آذین پورعسکر

۱

$(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+26!) \leadsto 7\times 6=42$

رقم یکان

$1!+3! \to 1+6=7$ ؛ $2!+4! \to 2+4=6$

چون ۵!×۲ یا همون صفر داره دیگه در یکان تأثیری نداره

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الف) $\sqrt[4]{(4+\sqrt7)^{-1}}\times\sqrt{1+\sqrt7}$

$\sqrt[2]{(4+\sqrt7)^{-\frac12}}\times\sqrt[2]{1+\sqrt7} \to \sqrt{(4+\sqrt7)^{-\frac12}(1+\sqrt7)}=\sqrt[4]{\frac{1}{4+\sqrt7}}\times\sqrt[4]{(1+\sqrt7)^2}=\sqrt[4]{\frac{1}{4+\sqrt7}\times\frac{8+2\sqrt7}{1}}=\sqrt[4]{\frac{2(4+\sqrt7)}{(4+\sqrt7)}}=\sqrt[4]{2}$

ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)=A \to A^2=\frac{2+5+2\sqrt2\sqrt5}{10+4+4\sqrt{10}}\times\frac{3-\sqrt5+3+\sqrt5-2(3-\sqrt5)(3+\sqrt5)}{1}=\frac{7+2\sqrt{10}\times(6-2(9-5))}{14+4\sqrt{10}}=\left|\frac{-2(7+2\sqrt{10})}{2(7+2\sqrt{10})}\right|=|-1|=1$

۳

الف) $\frac{\sqrt8+\sqrt{27}}{5-\sqrt6}-2(\sqrt[4]{9}-1)^{-1}=\frac{2\sqrt2+3\sqrt3}{5-\sqrt6}\times\frac{(5+\sqrt6)}{(5+\sqrt6)}=\frac{19(\sqrt2+\sqrt3)}{25-6}=\sqrt2+\sqrt3$ ؛ $\frac{2}{\sqrt3-1}\times\frac{\sqrt3+1}{\sqrt3+1}=\frac{2\sqrt3+2}{3-1}=\sqrt3+1 \leadsto \sqrt2+\sqrt3-\sqrt3-1=\sqrt2-1$

ب) $\frac{3\sqrt3-1}{4+\sqrt3}+\frac{1}{2-\sqrt3} \leadsto \frac{(3\sqrt3-1)(2-\sqrt3)+(4+\sqrt3)}{(4+\sqrt3)(2-\sqrt3)}=\frac{6\sqrt3-9-2+\sqrt3+4+\sqrt3}{8-4\sqrt3+2\sqrt3-3}=\frac{8\sqrt3-7}{5-2\sqrt3}\times\frac{5+2\sqrt3}{5+2\sqrt3}=\frac{(8\sqrt3-7)(5+2\sqrt3)}{25-12=13}=\frac{40\sqrt3+48-35-14\sqrt3}{13}=\frac{13(1+2\sqrt3)}{13}=2\sqrt3+1$

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الف) $A^2=\frac{1+\sqrt3+\sqrt3-1+2\sqrt{(\sqrt3+1)(\sqrt3-1)}}{\sqrt3-\sqrt2}-4=\frac{2\sqrt3+2\sqrt2}{\sqrt3-\sqrt2}-4 \to \frac{2(\sqrt3+\sqrt2)^2}{(\sqrt3-\sqrt2)(\sqrt3+\sqrt2)}=\frac{2(3+2+2\sqrt6)}{3-2}=10+4\sqrt6$ ؛ $\sqrt{\underbrace{10-4\sqrt6}_{4+6-4\sqrt6}}=\sqrt6-2+2 \to \sqrt6$

ب) $\sqrt[3]{2^2}\times 2\sqrt[4]{2}\times 2\sqrt[12]{2}=\sqrt[12]{2^8}\times\sqrt[12]{2^{15}}\times\sqrt[12]{2^{13}}=6\sqrt[12]{2^{12}}=6\times2=12$

162	2
81	3^4

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$\frac{3^n(1+3^1+3^2+3^3+3^4+3^5)}{2^n(2^{-2}+2^{-1}+1+2^1+2^2+2^3)}=52 \leadsto \frac{3^n(1+3+9+27+81+243)}{2^n\times\frac{3\times3\times7}{4}}=4\times13 \leadsto \frac{3^n(7\times4\times13)}{2^n\times\frac{3\times3\times7}{4}}=4\times13 \leadsto 3^n\times7=2^n\times\frac{3^2\times7}{4} \to 3^n\times2^2\times7=2^n\times3^2\times7$

$\left(\frac32\right)^n=\left(\frac32\right)^2 \to n=2$

$\frac14+\frac12+1+2+4+8=\frac{1+2+4+8+16+32}{4}=\frac{3\times3\times7}{4}$

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$a^2+b^2=n$ ؛ $(m-2ab)^2(m+2ab)^2=(m^2-4a^2b^2)^2=(a^4+b^4+2a^2b^2-4a^2b^2)^2$

$\to a=(\sqrt6-2)^{\frac14},\ b=(\sqrt6+2)^{\frac14}$

$t=9\sqrt6+10\sqrt2$

$t=\frac{2(\sqrt6-\sqrt2)}{2-\sqrt3}\times\frac{(2+\sqrt3)}{(2+\sqrt3)}=2(\sqrt6-\sqrt2)(2+\sqrt3)=\{9\sqrt6+10\sqrt2\}$ ؛ $t\cdot(a^2-b^2)^2=\left(\sqrt{\sqrt6-2}-\sqrt{\sqrt6+2}\right)^2$

$\sqrt6-2+\sqrt6+2=t(2-\sqrt3) \to 2\sqrt6-2\sqrt2=2(\sqrt6-\sqrt2)=t(2-\sqrt3)$

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$\left(a^2+\frac1{a^2}+2+2+(2a+\frac2a)\sqrt2\right)\left(a^2+\frac1{a^2}+2+2-(2a+\frac2a)\sqrt2\right)$

$\left(\frac{a^4+1}{a^2}+4+\frac{2a^2+2}{a}\right)=2t \to \left(a^2+\frac1{a^2}\right)^2=a^4+\frac1{a^4}+2=2^t \to 7-4\sqrt3+\frac{1}{7-4\sqrt3}+2=2^t$

$\frac{49+48-56\sqrt3+1}{7-4\sqrt3}=\frac{112-64\sqrt3}{7-4\sqrt3}=\frac{16(7-4\sqrt3)}{(7-4\sqrt3)}=16=2^4=2^t \to t=4$

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$\sqrt[15]{2^{10}}=\sqrt[3]{2^4}=\sqrt[3]{2^2}\times\sqrt[3]{2^4}$

$2\times 2^{3\times-\frac13}=\frac12 \longrightarrow A=2^2=4$

۹

$$\frac{۱}{a} - ۳ = \frac{۹}{\sqrt{۳}} - ۳ = \frac{۹ - ۳\sqrt{۳}}{\sqrt{۳}} \to ۳\sqrt{۳} - ۳$$

$$a^{\frac{۱}{۷}} = ۲۷ \times a^{\frac{۱۵}{۷}} = a^{\frac{۱۴}{۷}} = \frac{۱}{۲۷} \to a = \frac{\sqrt{۳}}{۹}$$

$$\frac{۳\sqrt{۳} - ۳}{\sqrt{۳}+۱} \times \frac{(\sqrt{۳}-۱)}{(\sqrt{۳}-۱)} = \boxed{۹ - ۳\sqrt{۳}}$$

۱۰

$$\left(\sqrt{x+a} - \sqrt{x-۴}\right)\left(\sqrt{x+a} + \sqrt{x-۴}\right) = -۱۰$$

$$\frac{x + a - x + ۴}{\sqrt{x+a} + \sqrt{x-۴}} = \to a+۴ \to ۲\sqrt{x+a} + \sqrt{x-۴}$$

$$= \frac{a}{۲} + ۲$$

$$\sqrt{x+a} + \sqrt{x-۴} - ۲ = \boxed{\frac{a}{۲}}$$

$$\sqrt[۷]{a} = ۲۷\left(\sqrt[۷]{a^{۱۵}}\right) \to \sqrt[۷]{a} = ۳^۳ \times a^۲\sqrt[۷]{a} \to \frac{۱}{۲۷} = a^۲ \to a = \sqrt[۲]{۳^{-۳}} \to \pm۳\sqrt{۳} \to +۳\sqrt{۳}$$

$$\frac{\frac{۱}{a} - ۳}{۱+\sqrt{۳}} = \frac{\frac{۱ - ۹\sqrt{۳}}{۳\sqrt{۳}}}{\frac{۱+\sqrt{۳}}{۱}} = \frac{۱ - ۹\sqrt{۳}}{(۳\sqrt{۳})(۱+\sqrt{۳})} = \frac{۱ - ۹\sqrt{۳}}{۳\sqrt{۳} + ۹\sqrt{۳}} \times \frac{\sqrt{۳}}{\sqrt{۳}} = \frac{\sqrt{۳} - ۲۷}{۹ + ۲۷} = \frac{\sqrt{۳} - ۲۷}{۳۶}$$