لعیا کبراسیان تمرین تام حد (۲) تکلیف ۲۲

۱-
$x^2-5x+2=0$ $\quad \frac{4\alpha+\beta^5}{5\beta^2}=\frac{4\alpha}{5\beta^2}+\frac{\beta^3}{5} \Rightarrow \frac{4}{5}\frac{\alpha}{\beta^2}=\frac{4}{5}\times\frac{\frac{5-\sqrt{17}}{2}}{\frac{25+17+10\sqrt{17}}{4_2}}=\frac{8}{5}\times\frac{5-\sqrt{17}}{42+10\sqrt{17}}$

$\alpha=\frac{5-\sqrt{17}}{2}$ $\quad \frac{380+350+92\sqrt{17}-92\sqrt{17}}{40}=\frac{73}{4}=\boxed{18,25}$

$\beta=\frac{5+\sqrt{17}}{2}$ $\quad \frac{\beta^3}{5}=\frac{1}{5}\times\frac{125+17\sqrt{17}+15\sqrt{17}(5+\sqrt{17})}{8}, \frac{1}{5}\times\frac{125+17\sqrt{17}+75\sqrt{17}+255}{8}$

$*\frac{8}{5}\times\frac{5-\sqrt{17}}{42+10\sqrt{17}}=\frac{8}{5}\times\frac{(5-\sqrt{17})(42-10\sqrt{17})}{42^2-1700}=\frac{8}{5}\times\frac{210-42\sqrt{17}-50\sqrt{17}+170}{64}=\boxed{\frac{380-92\sqrt{17}}{40}}$

$=\frac{1}{5}\times\frac{350+92\sqrt{17}}{8}=\boxed{\frac{350+92\sqrt{17}}{40}}$

۲-
$y=ax^2+bx+c$ $\quad (+3,-4)$ $\quad (-1,5,-4) \longleftarrow$ دو نقطه عرض های برابر دارند پس

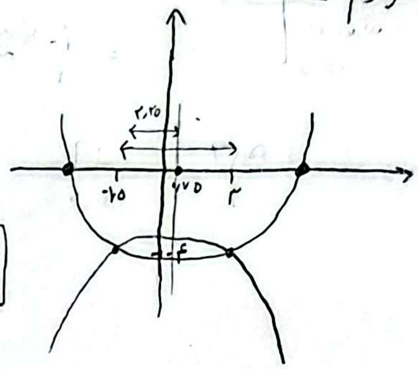
میانگین فاصله آنها برابر طول اکسترمم هست.

$|3+1,5|=4,5$

$4,5\div 2=\frac{9}{4}=2,25$

$2,25-1,5=0,75=\frac{3}{4}$ راس سهمی

$\frac{-b}{2a}=\frac{3}{4}\to\frac{b}{2a}=\frac{-3}{4}\to\boxed{\frac{b}{a}=S=\frac{-3}{2}}$



۳-
$x^2+2kx+5=0$ $\quad \frac{\sqrt{\Delta}}{|a|}=\frac{4}{3}k\to\frac{\sqrt{4k^2-20}}{1}=\frac{4}{3}k\longrightarrow(2\sqrt{k^2-5})3=4k$

$36(k^2-5)=16k^2$

$\left[\frac{k^2}{2}\right]=[4,5]=\boxed{4}$ $\quad 36k^2-180=16k^2$

$20k^2=180\longrightarrow k=\pm 3$

۴-
$\alpha^2+\beta^2=5$ $\quad y=a(x-h)^2+k\longrightarrow y=a(x-h)^2+1$ $\quad h=\frac{3-5}{2}=-1$ طول راس سهمی

عرض راس سهمی — طول راس سهمی — چون $(3, y)$ و $(-5, y)$ عرض یکسان دارند

$y=a(x+1)^2+1$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $\quad y=ax^2+2ax+a+1$ $\quad \frac{-b}{2a}=-1$ ، $\frac{-b}{a}=-2$

$5=4-2\alpha\beta$

$1=-2\alpha\beta$ $\quad c=$ محل برخورد با محور y ها $=a+1=\frac{-2}{3}+1$

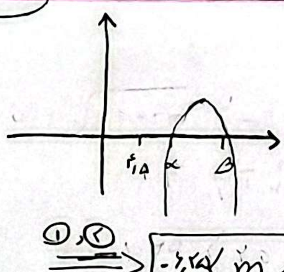
$-\frac{1}{2}=\alpha\beta\longrightarrow\frac{c}{a}=\frac{-1}{2}\longrightarrow\frac{a+1}{a}=\frac{-1}{2}\longrightarrow -a=2a+2$, $-2=3a$, $\frac{-2}{3}=a$ $\quad =\frac{1}{3}$ $\quad \boxed{(0,\frac{1}{3})}$ $\quad \boxed{\alpha+\beta=-2}$

۵-
$-x^2+5x+m=0$ $\quad \alpha,\beta<\frac{9}{2}$ $\quad$ ① $\Delta>0$

② $f(4,5)<0$ $\quad 25+4m>0$

$4m>-25$

$-\frac{81}{4}+\frac{45}{2}+m<0$ $\quad \boxed{m>-6,25}$



$m<-\frac{9}{4}\longrightarrow\boxed{m<-2,25}$ $\quad$ ①,② $\Longrightarrow \boxed{-6,25<m<-2,25}$

۶-
$y=mx^2-12x+5m-1$ $\quad \min\Big|_{m>0}+2\longrightarrow\frac{-\Delta}{4a}=\frac{-(144-\overbrace{4m(5m-1)}^{20m^2-4m})}{4m}=\frac{-144+20m^2-4m}{4m}$

$=\frac{4(-36+5m^2-m)}{4m}=2$

$\longrightarrow 2m=5m^2-36-m$

$0=5m^2-3m-36\to m=\frac{3\pm\sqrt{9+720}}{10}\xrightarrow{m>0}\frac{3+27}{10}=\boxed{3}$

$y=3x^2-12x+14\longrightarrow x_S=\frac{12}{6}=2\Longrightarrow x=2$ محور تقارن سهمی

۷-

$x^2+2(a+1)x+2a-1=0$

$\alpha\beta=a^2$

$\frac{c}{a}=a^2 \longrightarrow 2a-1=a^2$

$0=a^2-2a+1$

$0=(a-1)^2 \longrightarrow \boxed{a=1}$

۸-

$x^4-7x^2-5=0$

$x^2=t$

$t^2-7t-5=0$

$\left.\begin{aligned}\frac{-b}{a}&=S\\ \frac{c}{a}&=P\end{aligned}\right\}\ 2P^2-3PS+2S$

$\frac{-b}{a}=7 \qquad \frac{c}{a}=-5$

$50-3(7)(-5)+14=\boxed{169}$

۹-

$y=kx^2-4x-6 \sim x_s=\frac{4}{2k}=\frac{2}{k} \qquad y_s=\frac{-\Delta}{4a}=\frac{-(16+24k)}{4k}=\frac{-4}{k}-6$

① $y_s=\frac{-6k-4}{k}$

$y=-4x-4 \sim y_s=-4\times\frac{2}{k}-4$

② $y_s=\frac{-8}{k}-4 \sim y_s=\frac{-4k-8}{k}$

$①,② \Longrightarrow -4k-8=-6k-4$

$4=2k$

$\boxed{2=k} \longrightarrow y_s=\frac{-12-4}{2}=\boxed{-8}$

۱۰-

$y=-mx^2+mx+1 \qquad y=-m-x$

$-mx^2+mx+1\neq -m-x$

$-mx^2+mx+1+m+x\neq 0$

$-mx^2+x(m+1)+m+1\neq 0 \xrightarrow{\times -1} mx^2-x(m+1)-m-1\neq 0 \xrightarrow{\Delta<0} m^2+2m+1+4m^2+4m<0$

$5m^2+6m+1<0$

$(m+1)(m+\frac{1}{5})<0$

$-1 \qquad -\frac{1}{5}$

	-1		$-\frac{1}{5}$	
$+$	0	$-$	0	$+$

$(-1,-\frac{1}{5}) \sim$ ۰ عدد صحیح