تکلیف شمارهٔ ۲۲ / دهم دختر B — ۲۰ آفرین — شایان امامی

۱- $x^2-5x+2=0$ $\quad S=\frac{-b}{a}=5$ $\quad P=\frac{c}{a}=2 \Rightarrow \alpha\beta=2 \to \beta=\frac{2}{\alpha}$

$$\frac{4\alpha+\beta^5}{5\beta^2}=\frac{4\alpha}{5\beta^2}+\frac{\beta^3}{5}=\frac{4\alpha}{\frac{5\times4}{\alpha^2}}+\frac{\beta^3}{5}=\frac{\alpha^3+\beta^3}{5}=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{5}=\frac{125-3\times2\times5}{5}=\boxed{19}$$

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۲- $\begin{bmatrix}3\\-4\end{bmatrix}$ $\begin{bmatrix}-1.5\\-4\end{bmatrix}$ $\quad \frac{-b}{2a}=$ طول رأس $\rightsquigarrow \frac{3-1.5}{2}=0.75 \to \frac{-b}{2a}=\frac{1.5}{2} \to \frac{-b}{2}=\boxed{1.5}$

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۳- $x^2+2kx+5=0$ $\quad$ اختلاف ریشه‌ها $=|\alpha-\beta|=\frac{\sqrt{\Delta}}{|a|}=\frac{\sqrt{4k^2-20}}{1}=\frac{4}{3}k \to 2\sqrt{k^2-5}=\frac{4}{3}k$

$\to 3\sqrt{k^2-5}=2k \xrightarrow{\text{توان ۲}} 9(k^2-5)=4k^2 \to 5k^2-45=0 \to 5(k^2-9)=0$

$k^2-9=0 \to k^2=9$ $\quad \left[\frac{k^2}{2}\right]=\left[\frac{9}{2}\right]=[4.5]=\boxed{4}$

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۴- $\frac{-b}{2a}=\frac{3-5}{2} \Rightarrow \frac{-b}{a}=-2=S$ $\quad$ رأس $S(-1,1)$

$\alpha^2+\beta^2=S^2-2P=4-2P=5 \to 2P=-1 \to P=\frac{-1}{2}$

$y=a(x^2-Sx+P)=a(x^2+2x-\frac{1}{2})$

$x=-1 \to a(1-2-\frac{1}{2})=1 \to a=-\frac{2}{3} \to y=-\frac{2}{3}(x^2+2x-\frac{1}{2})$

اگر $x=0 \to y=\frac{-2}{3}(0+0-\frac{1}{2})=\boxed{\frac{1}{3}}$

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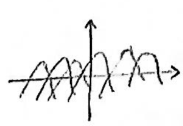
۵- $-x^2+5x+m=0$

$S=\frac{-b}{a}=5<\frac{9}{2}+\frac{9}{2}=9$



$P=\frac{c}{a}=-m>0 \to m<0$, $P<\frac{9}{2}\times\frac{9}{2}=\frac{81}{4} \to -m<\frac{81}{4} \to m>-\frac{81}{4}$

$\Delta>0 \to b^2-4ac>0 \to 25+4m>0 \to m>\frac{-25}{4}$

$\frac{9}{2}$ بزرگتر از ریشه $\to \frac{-81}{4}+\frac{45}{2}+m<0 \to \frac{-81+90}{4}+m<0 \to m<-\frac{9}{4}$

$\frac{-25}{4}<m<-\frac{9}{4}$

مقدارهای صحیح: $-3,-4,-5,-6$

$\boxed{4}$

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۶- $y_{min}=2$ $\quad$ شکل ∪ یعنی $a>0 \rightsquigarrow m>0$ $\qquad \frac{-\Delta}{4a}=2=\frac{-(144-20m^2+4m)}{4m} \longrightarrow$

$\frac{4(5m^2-m-36)}{4m}=2 \to 5m^2-m-36=2m \to 5m^2-3m-36=0$

ادامه صفحه بعد

5) $m^2 - 3m - 36 \quad (\times = -180) \longrightarrow (m-15)(m+12)$ → $m=-12$, $m=15$ $\xrightarrow{\div 5}$ $m = \frac{-12}{5}$ ✗ $\to a<0$, $m=3$ ✓

$y = 3x^2 - 12x + 14$ محور تقارن $= \frac{-b}{2a} \to \frac{12}{6} = 2 \to x = 2$

7- $\alpha, \alpha q, \beta$ $\xrightarrow{\text{واسطه هندسی}}$ $a^2 = \alpha\beta = P = \frac{c}{a} = \frac{2a-1}{1} \leadsto a^2 = 2a-1 \to a^2 - 2a + 1 = 0$

$\to (a-1)^2 = 0 \to a = 1$

8- $x^4 - 7x^2 - 5 = 0 \xrightarrow{x^2 = t} t^2 - 7t - 5 = 0 \quad \frac{7 \pm \sqrt{49+20}}{2}$ → $t = \frac{7+\sqrt{69}}{2}$ ✓ , $t = \frac{7-\sqrt{69}}{2}$ ✗ غیر قابل قبول

$t = x^2 = \frac{7+\sqrt{69}}{2} \longrightarrow x = \pm\sqrt{\frac{7+\sqrt{69}}{2}}$ → $S = 0$, $P = -\left(\frac{7+\sqrt{69}}{2}\right)$

$2P^2 - 3SP + 2S = 2P^2 = 2\frac{(7+\sqrt{69})^2}{4} = \frac{49+69+14\sqrt{69}}{2} = \frac{118+14\sqrt{69}}{2} = 59 + 7\sqrt{69}$

9- $y = kx^2 - 4x - 6 \longrightarrow S \begin{cases} \frac{-b}{2a} \to \frac{2}{k} \\ \frac{-\Delta}{4a} \to \frac{-(16+24k)}{4k} = \frac{-4-6k}{k} \end{cases}$

$y = -4x - 4$

$\leadsto -4\left(\frac{2}{k}\right) - 4 = \frac{-4-6k}{k} \longrightarrow \frac{-8-4k}{k} = \frac{-4-6k}{k} \to -8-4k = -4-6k$

$\longrightarrow 4 = 2k \to k = 2$ عرض رأس سهمی $= \frac{-4-6(2)}{2} = -8$

10- $\begin{cases} y = -mx^2 + mx + 1 \\ y = -m - x \end{cases}$ $-mx^2 + mx + 1 = -m - x \longrightarrow mx^2 + (-m-1)x - (m+1) = 0 \to \Delta < 0$

$(m+1)^2 + 4m(m+1) < 0$
$(m+1)(m+1+4m) < 0 \Rightarrow (m+1)(5m+1) < 0$

	-1		$-\frac{1}{5}$	
+		−		+

$m \in (-1, -\frac{1}{5})$

هیچ عدد صحیحی را شامل نمی‌شود