دیانا وطن‌خواه

۱۹/۵ آفرین

۱-

$\frac{4a+\beta^5}{5\beta^2} \Rightarrow \frac{4\alpha}{5\beta^2}+\frac{\beta^5}{5\beta^2} \to \frac{4\alpha}{5\beta^2}+\frac{\beta^3}{5}$

(۲)

$P=\alpha\beta=2 \to \beta=\frac{2}{\alpha}$

$S=\alpha+\beta=5 \quad \to \frac{4\alpha}{5(\frac{2}{\alpha})^2}+\frac{\beta^3}{5}=\frac{\alpha^3}{5}+\frac{\beta^3}{5} \xrightarrow{\alpha^3+\beta^3=S^3-3PS}$

$\frac{5^3-3\times2\times5}{5}=\frac{95}{5}=19$ ✓

$x_S=$ [crossed out]

$x_S=$

(۲)

مجموع صفرها $=\alpha+\beta=2x \to 2\times0/75=1/5$

$x=\frac{-b}{2a} \Rightarrow \alpha+\beta=\frac{-b}{a}$ ✓

$x=\frac{x_1+x_2}{2} \Rightarrow x=\frac{3+(-1/5)}{2}=0/75$

۳-

$|\alpha-\beta|=\frac{\sqrt{\Delta}}{|a|} \Rightarrow |\alpha-\beta|=\frac{4k}{3}$ (۲)

$x^2+2kx+5=0 \to S=-2k \quad P=5 \quad \Delta=4k^2-20 \quad \frac{4k}{3}=\frac{\sqrt{4k^2-20}}{1} \xrightarrow{\text{توان ۲}} \frac{16k^2}{9}=4k^2-20$

$\Rightarrow k^2-5=\frac{4k^2}{9} \to \frac{5k^2}{9}=5 \Rightarrow k^2=9 \to [\frac{k^2}{2}]=[\frac{9}{2}] \to [4/5] \Rightarrow [4]$ ✓

۴-

$x_v=\frac{3+(-5)}{2}=-1$ $\quad (x+1)^2=-\frac{3}{2}(y-1) \xrightarrow{x=0} (0+1)^2=-\frac{3}{2}(y-1)$ $\quad y=\frac{1}{3}$

$(x-x_v)^2=4p(y-y_v) \Rightarrow (x-(-1))^2=4p(y-1) \Rightarrow (x+1)^2=4p(y-1)$

$(3+1)^2=4p(y-1) \Rightarrow 16=4p(y-1) \Rightarrow y-1=\frac{4}{p} \xrightarrow{y=0} (x+1)^2=-4p \Rightarrow x+1=\pm\sqrt{-4p}$ ✓ (۲)

$\Rightarrow \alpha=-1+\sqrt{-4p},\ \beta=-1-\sqrt{-4p}$ / $x^2+2x+1+4p=0 \to \alpha+\beta=-2,\ \alpha\beta=1+4p$

$\alpha^2+\beta^2=5 \to 5=(-2)^2-2(1+4p) \Rightarrow 8p=2-5 \Rightarrow p=-\frac{3}{8}$ / $y-1=\frac{4}{p}=\frac{4}{-\frac{3}{8}} \Rightarrow y=-\frac{29}{3}$ $\quad A(3,-\frac{29}{3})$

$,B(-5,-\frac{29}{3})$

۵-

$\Delta\geq0 \Rightarrow 25+4m\geq0 \to m\geq-\frac{25}{4} \to m\geq-6$

$x=\frac{-5\pm\sqrt{25+4m}}{2}$ (۱/۵)

$x=\frac{-b}{2a}=2/5$

Δ باید مربع کامل باشد $\Leftarrow 25+4m=k^2$

$f(\frac{9}{2})=-(\frac{9}{2})^2+5(\frac{9}{2})+m \Rightarrow f(\frac{9}{2})=\frac{9}{4}+m \to \frac{9}{4}+m<0 \Rightarrow m<-\frac{9}{4} \to m\leq-3$

$m=-6 \to 25-24=1^2 \quad \alpha=\frac{-4}{2}=-2,\ \beta=\frac{-6}{2}=-3$ ✓ $\quad m=-3 \to 25-12=13$ ✗

$m=-5 \to 25-20=5$ ✗

$m=-4 \to 25-16=3^2$ ✓

بازای $-4, -6$

۶-
$y_s - 2 = \frac{-\Delta}{4a} = -\frac{(144 - 4m(5m-1))}{} \rightarrow 2m = -36 + 5m^2 - m \rightarrow$

$5m^2 - 3m - 36 \rightarrow \begin{cases} m = -12/5 \\ m = 3 \end{cases}$ ق
$5m^2 - m - 4 = 0$

$x_s = -\frac{b}{2a} = \frac{18}{9} = 2$ ۲

۷-
دنباله هندسی $\rightarrow \alpha , a , \beta \Rightarrow a^2 = \alpha\beta = 2a - 1 \rightarrow a^2 - 2a + 1 = 0$

$\alpha\beta = 2a - 1$ $(a-1)(a-1) \rightarrow a = 1$ ۲

۸-
$x^4 - 7x^2 - 5 = 0 \rightarrow y = x^2 \rightarrow y^2 - 7y - 5 = 0 \Rightarrow y_1 = \frac{7+\sqrt{69}}{2}, y_2 = \frac{7-\sqrt{69}}{2}$

$y \geq 0 \Rightarrow x_{1,2} = \pm\sqrt{\frac{7+\sqrt{69}}{2}}$ $y_2 = \frac{7-\sqrt{69}}{2}$ $\rightarrow x_1 = \sqrt{\frac{7+\sqrt{69}}{2}}, x_2 = -\sqrt{\frac{7+\sqrt{69}}{2}}$

$S = x_1 + x_2 = \left(\sqrt{\frac{7+\sqrt{69}}{2}}\right) + \left(-\sqrt{\frac{7+\sqrt{69}}{2}}\right) = 0$ $P = x_1 \cdot x_2 \Rightarrow \left(\sqrt{\frac{7+\sqrt{69}}{2}}\right)\left(-\sqrt{\frac{7+\sqrt{69}}{2}}\right) =$

$\frac{-7-\sqrt{69}}{2}$ $2P^2 - 3PS + 2S \xrightarrow{S=0} 2P^2 \Rightarrow 2\left(\frac{-7-\sqrt{69}}{2}\right)^2 = 59 + 7\sqrt{69}$ ۲

۹-
$y = kx^2 - 4x - 6$

$y = -4x - 4$ $y_s = k \times \frac{4}{k^2} - 4 \times \frac{2}{k} - 6 \rightarrow \frac{4}{k} - \frac{8}{k} - 6 \rightarrow \frac{-4}{k} - 6$

$x_s = \frac{-b}{2a} = \frac{4}{2k} = \frac{2}{k}$ $y = -4x - 4$ (روی خط) $\rightarrow \frac{-4}{k} - 6 = -4 \times \frac{2}{k} - 4 \rightarrow$

$\frac{4}{k} = \frac{2}{1} \Rightarrow k = 2$ ۲

$\frac{-4}{k} - 6 \Rightarrow \frac{-4}{2} - 6 = -2 - 6 = -8$

۱۰-
$-mx^2 + mx + 1 = -m - x \Rightarrow -mx^2 + (m+1)x + (1+m) = 0 \xrightarrow{\Delta < 0} \Delta = b^2 - 4ac \Rightarrow$

$\Delta = (m+1)^2 - 4(-m)(1+m) \Rightarrow (m+1)^2 + 4m(1+m) < 0 \rightarrow 5m^2 + 6m + 1 < 0 \Rightarrow$

$\Delta = 6^2 - 20 = 16 \rightarrow m = \frac{-6 \pm \sqrt{16}}{2 \times 5} \rightarrow m_1 = -0.2, m_2 = -1$

$-1 < m < -0.2$

~~عدد~~

بین ۱- و $-\frac{2}{10}$ عددی وجود ندارد.

۲ ارفاق

۵

$$x = \frac{-5 \pm \sqrt{25 - 4(-1)(m)}}{-2} < \frac{9}{2} \rightsquigarrow \pm\sqrt{25+4m} < 4$$

• $25+4m < 16 \rightsquigarrow -25 \leq 4m < -9 \rightsquigarrow -\frac{25}{4} \leq m < -\frac{9}{4} \rightsquigarrow m \in \{-6, -5, -4, -3\}$ (۴ عدد صحیح)

چون عبارت زیر رادیکال ≥ ۰ است باید حتماً باشد!