دیانا ناوش فرحان

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$$\frac{4\alpha+\beta^5}{5\beta^2} \Rightarrow \frac{4\alpha}{5\beta^2}+\frac{\beta^5}{5\beta^2} \to \frac{4\alpha}{5\beta^2}+\frac{\beta^3}{5}$$

$P = \alpha\beta = 2 \to \beta = \frac{2}{\alpha}$

$S = \alpha+\beta = 5$ $\quad \to \frac{4\alpha}{5\left(\frac{2}{\alpha}\right)^2}+\frac{\beta^3}{5} = \frac{\alpha^3}{5}+\frac{\beta^3}{5} \xrightarrow{\alpha^3+\beta^3 = S^3-3PS}$

$$\frac{5^3-3\times2\times5}{5} = \frac{95}{5} = 19$$

$x_s =$ ...

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$x = \frac{-b}{2a} \Rightarrow \alpha+\beta = \frac{-b}{a}$

$x = \frac{x_1+x_2}{2} \Rightarrow x = \frac{3+(-1.5)}{2} = 0.75$

مجموع صفرها $= \alpha+\beta-2.x \to 2\times0.75 = 1.5$

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$|\alpha-\beta| = \frac{\sqrt{\Delta}}{|a|} \Rightarrow |\alpha-\beta| = \frac{4k}{3}$

$x^2+2kx+5=0 \to S = -2k \quad P = 5 \quad \Delta = 4k^2-20$ $\quad \frac{4k}{3} = \frac{\sqrt{4k^2-20}}{1} \xrightarrow{\text{توان } 2} \frac{16k^2}{9} = 4k^2-20$

$\Rightarrow k^2-5 = \frac{4k^2}{9} \to \frac{5k^2}{9} = 5 \to k^2 = 9 \to \left[\frac{k^2}{2}\right] = \left[\frac{9}{2}\right] \to [4.5] \Rightarrow [4]$

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$x_v = \frac{3+(-5)}{2} = -1$ $\qquad (x+1)^2 = -\frac{3}{8}(y-1) \xrightarrow{x=0} (0+1)^2 = -\frac{3}{8}(y-1)$, $y = \frac{1}{3}$

$(x-x_v)^2 = 4p(y-y_v) \Rightarrow (x-(-1))^2 = 4p(y-1) \Rightarrow (x+1)^2 = 4p(y-1)$

$(3+1)^2 = 4p(y-1) \Rightarrow 16 = 4p(y-1) \Rightarrow y-1 = \frac{4}{p} \xrightarrow{y=0} (x+1)^2 = -4p \Rightarrow x+1 = \pm\sqrt{-4p}$

$\Rightarrow \alpha = -1+\sqrt{-4p},\ \beta = -1-\sqrt{-4p}$ / $x^2+2x+1+4p = 0 \to \alpha+\beta = -2,\ \alpha\beta = 1+4p$

$\alpha^2+\beta^2 = 5 \to 5 = (-2)^2-2(1+4p) \Rightarrow 8p = 2-5 \Rightarrow p = -\frac{3}{8}$ / $y-1 = \frac{4}{p} = \frac{4}{-\frac{3}{8}} \Rightarrow y = -\frac{29}{3}$ $\quad A(3, -\frac{29}{3})$, $B(-5, -\frac{29}{3})$

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$\Delta \geq 0 \Rightarrow 25+4m \geq 0 \to m \geq -\frac{25}{4} \to m \geq -6$ $\qquad x = \frac{5\pm\sqrt{25+4m}}{2}$

$x = \frac{-b}{2a} = 2/5$ $\qquad$ Δ باید مربع کامل باشد $\Leftarrow 25+4m = k^2$

$f(\frac{9}{2}) = -(\frac{9}{2})^2+5(\frac{9}{2})+m \Rightarrow f(\frac{9}{2}) = \frac{9}{4}+m \to \frac{9}{4}+m < 0 \Rightarrow m < -\frac{9}{4} \to m \leq -3$

$m = -6 \to 25-24 = 1^2$ $\qquad \alpha = \frac{4}{2} = 2,\ \beta = \frac{6}{2} = 3$ ✓ $\qquad m = -3 \to 25-12 = 13$ ✗

$m = -5 \to 25-20 = 5$ ✗

$m = -4 \to 25-16 = 3^2$ ✓

به ازای $-4, -6$

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$y_s = 2 = \frac{-\Delta}{4a} = -\frac{(144 - 4m(5m-1))}{4m} \rightarrow 2m = -34 + 5m^2 - m \rightarrow$

$5m^2 - 3m - 34 \rightarrow \begin{cases} m = -12/5 \\ m = 3 \quad \text{ق ق} \end{cases}$ $\qquad x_s = \frac{-b}{2a} = \frac{12}{6} = 2$

$5m^2 - m - 4 = 0$

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دنباله هندسی $\rightarrow \alpha, a, \beta \Rightarrow a^2 = \alpha\beta = 2a - 1 \rightarrow a^2 - 2a + 1 = 0$

$\alpha\beta = 2a - 1 \qquad (a-1)(a-1) \rightarrow \boxed{a = 1}$

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$x^4 - 7x^2 - 5 = 0 \rightarrow y = x^2 \rightarrow y^2 - 7y - 5 = 0 \Rightarrow y_1 = \frac{7 + \sqrt{69}}{2}, \; y_2 = \frac{7 - \sqrt{69}}{2}$

$y \geq 0 \Rightarrow x_{1,2} = \pm\sqrt{\frac{7 \pm \sqrt{69}}{2}} \qquad y_2 = \frac{7 - \sqrt{69}}{2} \rightarrow x_1 = \sqrt{\frac{7+\sqrt{69}}{2}}, \; x_2 = -\sqrt{\frac{7+\sqrt{69}}{2}}$

$S = x_1 + x_2 = \left(\sqrt{\frac{7+\sqrt{69}}{2}}\right) + \left(-\sqrt{\frac{7+\sqrt{69}}{2}}\right) = 0 \qquad P = x_1 \cdot x_2 \Rightarrow \left(\sqrt{\frac{7+\sqrt{69}}{2}}\right)\left(-\sqrt{\frac{7+\sqrt{69}}{2}}\right) =$

$\frac{-7 - \sqrt{69}}{2} \qquad 2P^2 - 3PS + 2S \xrightarrow{S = 0} 2P^2 \Rightarrow 2\left(\frac{-7 - \sqrt{69}}{2}\right) = \boxed{59 + 7\sqrt{69}}$

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$y = kx^2 - 4x - 6$

$y = -4x - 4 \qquad y_s = k \times \frac{4}{k^2} - 4 \times \frac{2}{k} - 6 \rightarrow \frac{4}{k} - \frac{8}{k} - 6 \rightarrow \frac{-4}{k} - 6$

$x_s = \frac{-b}{2a} = \frac{4}{2k} = \frac{2}{k} \qquad \underset{\text{روی خط}}{y = -4x - 4} \longrightarrow \frac{-4}{k} - 6 = -4 \times \frac{2}{k} - 4 \longrightarrow$

$\frac{4}{k} = \frac{2}{1} \Rightarrow \boxed{k = 2}$

$\frac{-4}{k} - 6 \Rightarrow \frac{-4}{2} - 6 = -2 - 6 = \boxed{-8}$

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$-mx^2 + mx + 1 < -m - x \Rightarrow -mx^2 + (m+1)x + (1+m) < 0 \xrightarrow{\Delta < 0} \Delta = b^2 - 4ac \Rightarrow$

$\Delta = (m+1)^2 - 4(-m)(1+m) \Rightarrow (m+1)^2 + 4m(1+m) < 0 \rightarrow 5m^2 + 6m + 1 < 0 \Rightarrow$

$\Delta = 6^2 - 20 = 16 \rightarrow m = \frac{-6 \pm \sqrt{16}}{2 \times 5} \rightarrow m_1 = -0.2, \; m_2 = -1$

$-1 < m < -0.2$

[illegible]