

$$x_0 = -\frac{b}{2a} = -1 \Rightarrow b = 2a \quad y_0 = 9 = a(-1)^2 + (-1)b + c \Rightarrow c = 9 + a$$

$$(3,1) \quad 1 = a(3)^2 + b(3) + c = 9a + 3(2a) + (9+a) = 14a = -1 \Rightarrow a = -\frac{1}{14}$$

$$\Rightarrow b = 2a = -\frac{1}{7}, \quad c = 9 + a = \frac{127}{14} \Rightarrow y = -\frac{1}{14}x^2 - \frac{1}{7}x + \frac{127}{14}$$

$$\Delta > 0 \Rightarrow m^2 - 1(m+4) > 0 \Rightarrow m^2 - 1m - 4 > 0 \Rightarrow (m+4)(m-1) > 0$$

$$S = -\frac{b}{a} = -\frac{m}{1} > 0 \Rightarrow m < 0$$

$$P = \frac{c}{a} = \frac{m+4}{1} > 0 \Rightarrow m > -4$$

$$\Rightarrow -4 < m < 0$$

$$S = \frac{1-2m}{3} \quad P = \frac{1-m}{3} \Rightarrow S = \frac{1}{P} \Rightarrow \frac{1-2m}{3} = \frac{3}{1-m}$$

$$(1-2m)(1-m) = 9 \Rightarrow 2m^2 - 3m + 2 = 9 \Rightarrow 2m^2 - 3m - 7 = 0$$

$$\Delta = (3)^2 - 4(2)(-7) = 121 \Rightarrow m = \frac{3 \pm 11}{4} \Rightarrow m = \frac{14}{4} = \frac{7}{2} \quad \text{or} \quad m = -\frac{8}{4} = -2$$

$$x^2 - x - 4 = 0 \Rightarrow \Delta = 17 \Rightarrow x_1 = \frac{1 + \sqrt{17}}{2}, \quad x_2 = \frac{1 - \sqrt{17}}{2}$$

$$\Rightarrow P_{y_{1,2}} = \frac{(x_1^2 + 1)(x_2^2 + 1)}{P} = \frac{x_1^2 x_2^2 + x_1^2 + x_2^2 + 1}{P} = \frac{(x_1 x_2)^2 + 4x_1^2 - 4x_2^2 + 1}{P}$$

$$= \frac{-4(-4) + 1}{-4} = \frac{17}{-4} = -\frac{17}{4} \quad \text{or} \quad \Delta_{y_1, y_2} = \frac{17}{4} \Rightarrow y^2 + \frac{17}{4}y + \frac{17}{4} = 0$$

$$\Rightarrow 4y^2 + 17y + 17 = 0$$

$$t = \sqrt{x} \Rightarrow (t^2 + \frac{1}{t^2} + 1)(t^2 - 1) = 2t \Rightarrow (t^2 - 1)(t^2 + t^2 + 1) = 2t^3 - 1$$

$$\Rightarrow t^4 - 1 = 2t^3 \Rightarrow t^4 - 2t^3 - 1 = 0 \xrightarrow{y=t^3} y^{\frac{4}{3}} - 2y - 1 = 0 \Rightarrow x = 1 \pm \sqrt{2}$$

$$\Delta = \frac{1}{4}$$

$$r, r \Rightarrow r^2 = \frac{2}{p} \rightarrow r^2 = \frac{2}{q} \Rightarrow r = \pm \frac{\sqrt{2}}{p}$$

$$\delta \Rightarrow r = \frac{a}{p} \rightarrow a = \frac{r}{p} \rightarrow \frac{a}{p} = \frac{1}{p} \rightarrow a = 1$$

$$a = -\frac{r}{p} \rightarrow a = -1 \rightarrow 1 - (-1) = \underline{2}$$

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$$x(ax + 3 + 2a) = 0 \xrightarrow{x=0} x = \frac{-3-2a}{2} \xrightarrow{\text{ریشه دوم منفی نباشد}} \frac{-3-2a}{2} > 0$$

$$\rightarrow \underline{\frac{-3}{2} \leq a < 9}$$

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$$x\delta_1 = -\frac{a}{p} \quad x\delta_2 = -1 \rightarrow -\frac{a}{p} = -1 \rightarrow a = p$$

$$y=1 \rightarrow 1 = x^2 + 2x - 2 \rightarrow x^2 + 2x - 3 = 0, \quad 1 = -x^2 - 2x + b \rightarrow x^2 + 2x + (1-b) = 0$$

$$\Delta > 0 \quad \checkmark \text{ دو ریشه دارد}$$

$$\rightarrow \Delta = -2 - 4(1-b) > 0 \rightarrow 4b > 6 \rightarrow b > \frac{3}{2} \Rightarrow \text{دو ریشه ای غیر منفی} = 3 \rightarrow ab = 2, 3 = \underline{6}$$

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$$pax^2 + ax - 4 \neq 0 \rightarrow \delta = \frac{-a}{pa} = -\frac{1}{p} \rightarrow p, \frac{-4}{pa} = -\frac{p}{a}$$

$$rx^2 - ax + b = 0 \rightarrow \delta = \frac{a}{p} \quad r = \text{ریشه} \rightarrow r + \frac{1}{p}r + \frac{1}{p} \rightarrow (r + \frac{1}{p})(r + \frac{1}{p})$$

$$\rightarrow \frac{r}{-1} + r + 1 = \frac{a}{p} \rightarrow a = 1 \rightarrow (r + \frac{1}{p})(r + \frac{1}{p}) = \frac{p}{p} + \frac{1}{p}\frac{1}{p} + \frac{1}{p} = \frac{p^2 + 1 + p}{p^2}$$

$$\rightarrow \frac{b}{p} = -3 \rightarrow b = -9 \rightarrow \frac{ab}{p} = \frac{1 \times -9}{p} = -\frac{9}{p}$$

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$$\begin{cases} x^2 + 4x + m = 0 \\ -x^2 + 2x - 3m = 0 \end{cases} \Rightarrow 5x + 7m = 0 \rightarrow x = -\frac{7m}{5}$$

$$\rightarrow m^2 - 9m + m = 0 \rightarrow m(m-8) = 0 \rightarrow x = -8 \Rightarrow x^2 + 2x - 16 = 0$$

$$(x-3)(x+5)$$

$$x^2 + 4x + 5 = 0 \rightarrow (x+1)(x+5) \Rightarrow 5 - (-1) = \underline{6}$$

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