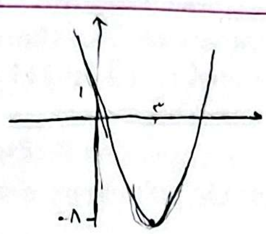
-۱

الف) $y = 2x^2 - 4x + 1 \quad \min \left| \begin{matrix} 1 \\ -1 \end{matrix} \right.$

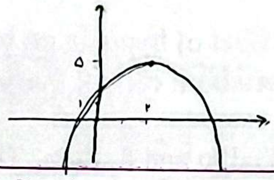
ب) $y = -2x^2 + 3x - 5 \quad \max \left| \begin{matrix} \frac{3}{4} \\ -\frac{9}{8} + \frac{9}{4} - 5 = \frac{9}{8} - 5 = \frac{-31}{8} \end{matrix} \right.$

-۲

الف) $y = x^2 - 6x + 1$



ب) $y = -x^2 + 4x + 1$



-۳

$4x^3 + kx^2 - 9x - 2 = 0 \xrightarrow{x=-1} -4 + k + 9 - 2 = 0 \qquad \boxed{k = -3}$

$0 = x^2 - Sx + P$

$0 = x^2 - x - 2 \rightarrow -1 ,\ +2$

-۴

$x^2 - 3mx + m = 0 \qquad \sqrt{\alpha} - \sqrt{\beta} = 1$

$\alpha + \beta - 2\sqrt{\alpha\beta} = 1$

$3m - 2\sqrt{m} = 1 \xrightarrow{\sqrt{m} = t} 3t^2 - 2t - 1 = 0 \rightarrow t = \sqrt{m} = \textcircled{1} \rightarrow m = 1$

$t = \sqrt{m} \neq -\frac{1}{3}\ \times$

$2x^2 - mx - m = 0$

$P = \frac{-m}{2} = \boxed{-\frac{1}{2}}$

-۵

$y = 2x^2 - (m+2)x + m$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$(\alpha - \beta)^2 = \frac{(m+2)^2}{4} - 2m$

$(\alpha - \beta)^2 = \frac{m^2 + 4 + 4m - 8m}{4} \rightarrow \alpha - \beta = \frac{m-2}{2}$

$S = \frac{(\alpha - \beta)h}{2} = \frac{3}{4} \rightarrow (\alpha - \beta)h = \frac{3}{2}$

$\frac{m-2}{2} \times m = \frac{3}{2}$

$m^2 - 2m - 3 = 0 \rightarrow \textcircled{-1} ,\ \textcircled{3}$

$y = x^2 - mx + 1$

$\frac{-b}{2a} = \frac{-m}{2} \rightarrow \boxed{\frac{1}{2}} ,\ \boxed{-\frac{3}{2}}$

-۶

۱ مقدار

$y = ax^2 + 3x + a$ — کمترین مقدار مثبت $\rightarrow a > 0$

$\min \left| \frac{-\Delta}{4a} = \frac{7}{8} \right. \rightarrow -2\Delta = 7a$

$\Delta = \frac{-7a}{2}$

$9 - 4a^2 = \frac{-7a}{2}$

$0 = 4a^2 - \frac{7a}{2} - 9$

$0 = 8a^2 - 7a - 18$

$a = \frac{7 \pm \sqrt{49 + 576}}{16} = \frac{7 \pm 25}{16} \rightarrow 2\ ✓ ,\ \frac{-9}{8}\ \times$

-۷

$x^2 - (a+1)x + a = 0 \longrightarrow 1 ,\ a \to 3$ } ③

$\alpha ,\ \alpha + 2$

$x^2 - (3a+1)x + b = 0 \xrightarrow{a=3} x^2 - 10x + b = 0$

$\beta ,\ \beta + 2$

$x = \frac{10 \pm \sqrt{100-4b}}{2} \rightsquigarrow \frac{10 - \sqrt{100-4b}}{2} + \frac{4}{2} = \frac{10 + \sqrt{100+4b}}{2}$

$10 - \sqrt{100-4b} + 4 = 10 + \sqrt{100+4b}$

$4 = 2\sqrt{100+4b}$

$2 = \sqrt{100+4b}$

$4 = 100 + 4b$

$-24 = \frac{-96}{4} = b$

$P = b = \boxed{-24}$

$|3 - (-24)| = \boxed{27}$

-۸

$y = -ax^2 + ax + 2 \quad ext \Big| \frac{1}{2}$; $-\frac{a}{4} + \frac{a}{2} + 2 = \left(\frac{a}{4} + 2\right)$

$y = 2bx^2 - bx - 1 \quad ext \Big| \frac{1}{4}$; $\frac{b}{8} - \frac{b}{4} - 1 = \left(\frac{-b}{8} - 1\right)$

$\Longrightarrow \frac{-b}{8} - 1 = \frac{-a}{16} + \frac{a}{4} + 2$

$-3 = \frac{3a}{16} + \frac{2b}{16} \longrightarrow 3a + 2b = -48$

$2b = -12$

$b = -6$

$\Longrightarrow \frac{a}{4} + 2 = 2b \times \frac{1}{4} - \frac{b}{2} - 1$

$\frac{a}{4} = -3 \longrightarrow a = -12$

$b - a = -6 + 12 = \boxed{6}$

-۹

$y = 25\alpha x^2 + 4x + \beta \longrightarrow -5x^2 + 4x + 1$

$max \Big| \frac{+2}{5}$; $-5 \times \frac{4}{25} + \frac{8}{5} + 1 = \frac{9}{5}$ } 1 [illegible]

$\alpha\beta = \frac{\beta}{25\alpha} \to \beta\left(\alpha - \frac{1}{25\alpha}\right) = 0$

$\alpha \neq 0 ,\ \beta > \alpha$ چون $\neq 0$

$\alpha = \frac{1}{25\alpha} \to 25\alpha^2 = 1$

$\alpha = \boxed{\pm \frac{1}{5}}$

$\alpha + \beta = \frac{-4}{25\alpha} \xrightarrow{\alpha = \frac{1}{5}} \beta = -1$ ✗

$\xrightarrow{\alpha = -\frac{1}{5}} \boxed{\beta = 1}$ ✓ $\rightarrow \boxed{\alpha = -\frac{1}{5}}$

۱۰-

$x^2 - (a^2 + b^2 - 12)x + a + b - 1 = 0$

$a + b = a^2 + b^2 - 12 \xrightarrow{a^2+b^2 = (a+b)^2 - 2ab} a + b = (a+b)^2 - 2ab - 12$

$\xrightarrow{ab = a+b-1} a + b = (a+b)^2 - 2(a+b-1) - 12$

$S = S^2 - 2(S-1) - 12$

$0 = S^2 - 3S - 10 \longrightarrow S = -2$ ✗

$\searrow S = 5$ ✓