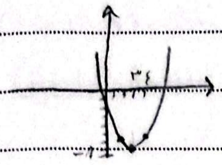
۱- الف) $y=2x^2-4x+1$ min $\left| x_s=\frac{-b}{2a}=\frac{+4}{4}=1 \quad y_s=\frac{-\Delta}{4a}=-1 \right.$

ب) $y=-2x^2+3x-5$ max $\left| x_s=\frac{-b}{2a}=\frac{-3}{-4}=+\frac{3}{4} \quad y_s=\frac{-\Delta}{4a}=\frac{+31}{-8} \right.$

۲- الف) x^2-6x+1 min $\left| \frac{-b}{2a}=\frac{+6}{2}=3 \quad \frac{-\Delta}{4a}=-8 \right.$

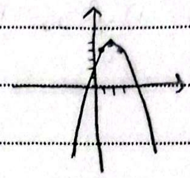


x	y
2	-7
3	-8
4	-7

ب) $y=-x^2+4x+1$

x	y
1	4
2	5
3	4

$x_s=\frac{-b}{2a}=\frac{-4}{-2}=+2$ max $y_s=5$



۳-

$4x^3+Kx^2-9x-2=0 \quad \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=1+4=5,\ \alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=1+6=7$

$4\alpha^3+K\alpha^2-9\alpha-2=0$

$4\beta^3+K\beta^2-9\beta-2=0$ $\Big\}\ \underbrace{4\alpha^3+4\beta^3}_{28}+\underbrace{K\alpha^2+K\beta^2}_{5K}\underbrace{-9\alpha-9\beta}_{-9}-4=0 \Rightarrow 28+5K-13=0 \to 5K=-15 \to \boxed{K=-3}$

۴-

$x^2-3mx+m=0\ ,\ 2x^2-mx+m=0 \xrightarrow{m=1} \frac{c}{a}=\left(\frac{-1}{2}\right),\ \xrightarrow{m=\frac{1}{9}} \left(\frac{-1}{18}\right)=\frac{c}{a}$

$|\sqrt{\alpha}-\sqrt{\beta}|=1 \to (\sqrt{\alpha}-\sqrt{\beta})^2=1 \to \alpha+\beta-2\sqrt{\alpha\beta}=1 \to 3m-2\sqrt{m}=1$

$\alpha+\beta=3m,\ \alpha\beta=m \to (3m-1)^2=(2\sqrt{m})^2 \to 9m^2-6m+1=4m \to 9m^2-10m+1=0$

$m=\frac{10\pm\sqrt{100-36}}{18}=\frac{10\pm8}{18} \quad m<\begin{matrix}1\\ \frac{1}{9}\end{matrix} \qquad 2x^2-mx-m=0 \to \frac{c}{a}=\frac{-m}{2}$

۵-

$y=2x^2-(m+3)x+m$

$\alpha+\beta=\frac{m+3}{2},\ \alpha\beta=\frac{m}{2}$

S مثلث $:\frac{1}{2}|\beta-\alpha|\times|m|=\frac{3}{4} \to \frac{1}{2}\times\left|\frac{m-3}{2}\right|\times|m|=\frac{3}{4} \to |m(m-3)|=3 \Rightarrow m<\begin{matrix}3\\ -1\end{matrix}$

$|\beta-\alpha|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\frac{|m-3|}{2}$

$y=x^2-mx+1,\ x_s=\frac{m}{2} \qquad x<\begin{matrix}\left(\frac{3}{2}\right)\\ \left(\frac{-1}{2}\right)\end{matrix}$

۶-

$y=ax^2+3x+a,\ a>0$

$y_s=\frac{-\Delta}{4a}=\frac{4a^2-9}{4a}=\frac{7}{8} \to 8(4a^2-9)=28a \to 32a^2-28a-72=0 \to 8a^2-7a-18=0$

$\Delta=49+576=625 \to a=\frac{7\pm25}{16} \to a<\begin{matrix}(2)\ ✓\\ \frac{-9}{8}\ \times\end{matrix}$ غق

(یک مقدار)

۷-

$x^2+(a+1)x+a=0$ ، ضریب $x=\frac{-b}{a} \to m+(m+2)=2m+2=-(a+1) \to 2m+2=-(a+1)$

$c=a=\frac{c}{a} \qquad m(m+2)=a \to m=-\frac{(a+3)}{2}$ ، ریشه ها $m, m+2$ باشد

$-\frac{(a+3)}{2}\times\left(\frac{-(a+3)}{2}+2\right)=a \to a^2-2a-3=0 \to a=3,\ x^2+4x+3=0 \to x<\begin{matrix}-1\\ -3\end{matrix}$

$x^2+(2a+1)x+b=0 \to x^2+10x+b=0 \to b=4\times6=24$

$\Rightarrow 24-3=\boxed{21}$

۸- $y=-ax^2+ax+2 \longrightarrow \left(\frac{-b}{a}=\frac{-a}{14}+\frac{a}{4}+2\right)\times 14$, $\frac{-b}{2a}=\frac{-a}{-2a}=\frac{1}{2}$, $\frac{-\Delta}{4a}=\frac{-a-8}{4}$

$y=2bx^2-bx-1 \longrightarrow \frac{-b}{2a}=\frac{+b}{4b}=\frac{1}{4}$, $\frac{-\Delta}{4a}=\frac{-b^2+8b}{8b}=\frac{-b+8}{8}$, $\frac{-a-8}{4}=\frac{b}{4}-\frac{b}{2}-1=-1$

$$\begin{cases} -2b-14=-a+4a+22 \longrightarrow -2b-2a=48 \longrightarrow -2b-24=48 \longrightarrow -2b=72 \longrightarrow b=-18 \\ -a-8=-4 \longrightarrow -a=4 \longrightarrow a=-4 \end{cases}$$

$b-a=-18+4=-14$

۹- $2\omega\alpha(x-\alpha)(x-\beta)=0 \longrightarrow 2\omega\alpha x^2-2\omega\alpha(\alpha+\beta)x+2\omega\alpha^2\beta=0$

$2\omega\alpha^2\beta=\beta \Rightarrow 2\omega\alpha^2=1 \Rightarrow \alpha=\pm\frac{1}{\omega}$

$-2\omega\alpha^2-2\omega\alpha\beta=4 \Rightarrow \alpha\beta=\frac{-1}{\omega} \longrightarrow \beta>\alpha \longrightarrow \alpha=\frac{-1}{\omega}, \beta=1$

$y=2\omega\alpha x^2+4\alpha+\beta \Rightarrow -\omega x^2+4x+1$

$x_s=\frac{-b}{2a}=\frac{-(+)}{-}=+$
$y_s=\frac{-\Delta}{4a}=\frac{-(+)}{-}=+$ } رأس سهمی در ربع اول

۱۰- $x^2-(a^2+b^2-12)x+a+b-1=0$

$\frac{c}{a}=\frac{a+b-1}{1}=a+b-1=ab$

$\frac{-b}{a}=-a^2+b^2+12=a+b$

$(a+b)^2=a^2+b^2+2ab$

$(a+b)^2=a+b+12+2a+2b-2$

$(a+b)^2=3a+3b+10 \longrightarrow (a+b)^2=3(a+b)+10$

$a+b=t$

$t^2=3t+10 \longrightarrow t^2-3t-10=0$

$(t-5)(t+2)=0$ ← $t=5$ ✓ , $t=-2$ ✗