گروه: دهم یکشنبه — سری: ۲۴ — نام و نام خانوادگی: تانیا ترک ترابی

۱۹ آذر

1
الف) $y = 2x^2 - 4x + 1$ $\quad$ min $\left| \begin{array}{l} -\frac{b}{2a} = \frac{-(-4)}{4} = 1 \\ y = 2 - 4 + 1 = -1 \end{array} \right.$ ✓

۱٫۷۵

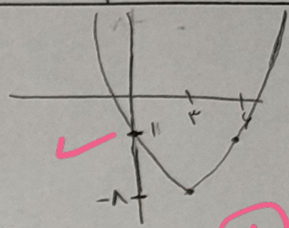
ب) $y = -2x^2 + 3x - 5$ $\quad$ max $\left| \begin{array}{l} -\frac{b}{2a} = -\frac{3}{-4} = \frac{3}{4} \\ y = -2\left(\frac{3}{4}\right) + 3\left(\frac{3}{4}\right) - 5 = -\frac{3}{2} + \frac{9}{4} - 5 = -\frac{23}{4} \end{array} \right.$ ✗

$-2\left(\frac{9}{16}\right) + 3\left(\frac{3}{4}\right) - 5$

$-\frac{9}{8} + \frac{9}{4} - 5 = -\frac{31}{8}$

2
الف) $y = x^2 - 6x + 1$ $\quad$ min $\left| \begin{array}{l} -\frac{b}{2a} = 3 \\ y = 9 - 18 + 1 = -8 \end{array} \right.$

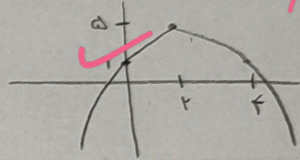
x	0	3	6
y	1	-8	1



۲

ب) $y = -x^2 + 4x + 1$ $\quad$ max $\left| \begin{array}{l} -\frac{b}{2a} = 2 \\ y = -4 + 8 + 1 = 5 \end{array} \right.$

x	0	2	4
y	1	5	1



3
$4x^3 + kx^2 - 9x - 2 = 0 \Rightarrow \underbrace{(x-\alpha)(x-\beta)}_{x^2 - (\alpha+\beta)x + \alpha\beta = x^2 - x - 2}(4x + t) \Rightarrow (x^2 - x - 2)(4x + t) \Rightarrow$

$(x^2)(4x + t) - (x)(4x + t) - 2(4x + t) = 4x^3 + (t - 4)x^2 + (-t - 8)x - 2t = 4x^3 + kx^2 - 9x - 2$

$k = t - 4 \quad , -9 = -t - 8 \quad , -2 = -2t \Rightarrow t = 1 \qquad k = \underset{1}{t} - 4 \Rightarrow \boxed{k = -3}$

۲

4
$x^2 - 3mx + m = 0$ $\qquad |\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = 1 \Rightarrow (1)^2 = \left(\sqrt{9m^2 - 4m}\right)^2 \Rightarrow 9m^2 - 4m - 1 = 0$

$m = \frac{4 \pm \sqrt{16 + 36}}{18} = \frac{2 \pm \sqrt{13}}{9}$

$\alpha \times \beta = \frac{c}{a} = m \quad , \alpha + \beta = -\frac{b}{a} = 3m$

۱٫۵

$2x^2 - mx - m = 0$

$\alpha \times \beta = \frac{c}{a} = -\frac{m}{2} \Rightarrow -\frac{2 + \sqrt{13}}{18}$ یا $-\frac{2 - \sqrt{13}}{18}$

5
$2x^2 - (m + 2)x + m = 0 \quad \left\{ \begin{array}{l} x_1 + x_2 = \frac{m+2}{2} \\ x_1 x_2 = \frac{m}{2} \end{array} \right.$ $\qquad A(x_1, 0) / B(x_2, 0) / C(0, m) \leftarrow$ نقاط رئوس مثلث

$S = \frac{1}{2}|x_1 - x_2| \times |m| = \frac{3}{4} \qquad |x_1 - x_2| = \frac{\sqrt{(m+2)^2 - 8m}}{2} = \frac{|m-2|}{2} \qquad \frac{1}{2} \times |m| \times \frac{|m-2|}{2} = \frac{3}{4} \Rightarrow$

$|m| \times |m - 2| = 3$

① $m > 2 \Rightarrow m(m - 2) = 3 \Rightarrow m^2 - 2m - 3 = 0 \Rightarrow (m - 3)(m + 1) = 0 \xrightarrow{m > 2} \boxed{m = 3}$

② $0 < m < 2 \Rightarrow m(2 - m) = 3 \Rightarrow m^2 - 2m + 3 = 0 \longrightarrow \Delta < 0$ ③ $m < 0 \Rightarrow (-m)(2 - m) = 3 \Rightarrow m^2 - 2m - 3 = 0 \xRightarrow{(m-3)(m+1)} m = -1$

ادامه سوال در صفحه بعد است

$m = 3$ یا $m = -1$ $\quad y = -x^2 - mx + 1 \quad x_{\text{راس}} = \frac{m}{2} \quad y_{\text{راس}} : (\frac{m}{2})^2 - m(\frac{m}{2}) + 1 = 1 - \frac{m^2}{4}$ ۵

① $m = 3 \to y_{\text{راس}} = 1 - \frac{9}{4} = \boxed{-\frac{5}{4}}$ $\quad$ ② $m = -1 \to y_{\text{راس}} = 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$

۱/۵

$y = ax^2 + 3x + a \qquad a = \frac{7 \pm \sqrt{625}}{2 \times 8} = \frac{7 \pm 25}{16} \quad a_1 = 2 \;, a_2 = -\frac{9}{8}$

min شرط $\to a > 0 \Rightarrow \boxed{a = 2}$

min $\left| \begin{array}{l} -\frac{b}{2a} = -\frac{3}{2a} \\ a(\frac{9}{4a^2}) + 3(-\frac{3}{2a}) + a = \frac{9}{4a} - \frac{9}{2a} + a = \frac{9 - 18 + 4a^2}{4a} = \frac{4a^2 - 9}{4a} = \frac{7}{8} \Rightarrow \end{array} \right.$ ۶

۲

$28a = 32a^2 - 72 \Rightarrow 32a^2 - 28a - 72 = 0 \Rightarrow 8a^2 - 7a - 18 = 0 \quad \Delta = 49 + 576 = 625$

$x^2 - (a+1)x + a = 0 \to \alpha + \beta = a + 1 \;, \alpha \times \beta = a \qquad \alpha = 2n - 1 \;, \beta = 2n + 1$

$(2n-1) + (2n+1) = a + 1 \Rightarrow a = 4n - 1$ ☆ $\qquad (2n-1)(2n+1) = a \Rightarrow 4n^2 - 1 = a$ ☆☆

☆,☆☆ $\to 4n \cancel{-1} = 4n^2 \cancel{-1} \Rightarrow 4n(n-1) = 0 \xrightarrow{n \geq 1} n = 1 \to a = 4n - 1 \Rightarrow a = 3 \quad \boxed{\overset{2-1}{\alpha = 1}} \;, \boxed{\overset{2+1}{\beta = 3}}$

$P_1 = \alpha \times \beta = 3 \;|\; x^2 - (3a+1)x + b = 0 \xrightarrow{a=3} x^2 - 10x + b = 0 \to p + q = 10 \;, p \times q = b$ ۷

$p = 2k \;, q = 2k + 2 \qquad 2k + (2k+2) = 10 \Rightarrow 4k + 2 = 10 \Rightarrow k = 2 \qquad p = 2(2) = 4 \quad q = 2(2) + 2 = 6$

$P_2 = pq = 24 \qquad P_2 - P_1 = 24 - 3 = 21$

$y = -ax^2 + ax + 2 \left| \begin{array}{l} \frac{a}{2a} = \frac{1}{2} \\ y = -a(\frac{1}{2})^2 + a(\frac{1}{2}) + 2 = \frac{a}{4} + 2 \end{array} \right.$

$y_{v_1} = 2b(x_{v_1})^2 - b(x_{v_1}) - 1 \Rightarrow$

$\frac{a}{4} + 2 = 2b(\frac{1}{2})^2 - b(\frac{1}{2}) - 1 \Rightarrow \frac{a}{4} + 2 = -1$

$\frac{a}{4} = -1 - 2 \Rightarrow \boxed{a = -12}$ ۸

$x_2 = -\frac{-b}{4b} = \frac{1}{4}$ ☆

$y_2 = 2b(\frac{1}{4})^2 - b(\frac{1}{4}) - 1 \Rightarrow y_2 = -\frac{b}{8} - 1 = -\overset{-12}{a}(\frac{1}{4})^2 + \overset{-12}{a}(\frac{1}{4}) + 2 \Rightarrow b = -6$

۲

$b - a = (-6) - (-12) = 6$

$\alpha + \beta = -\frac{4}{25\alpha} \Rightarrow \alpha(\alpha + \beta) = -\frac{4}{25} \Rightarrow \alpha^2 + \alpha\beta = -\frac{4}{25}$ ☆ $\qquad \alpha \times \beta = \frac{\beta}{25\alpha} \xrightarrow{\div \beta} \alpha = \frac{1}{25\alpha} \Rightarrow$

$25\alpha^2 = 1 \Rightarrow \alpha = \pm \frac{1}{5}$

$\alpha\beta = \frac{\beta}{25 \times \frac{1}{5}} = \frac{\beta}{5}$

① $\alpha^2 + \alpha\beta = -\frac{4}{25} \Rightarrow (\frac{1}{5})^2 + \frac{\beta}{5} = -\frac{4}{25} \Rightarrow \beta = -1 \;, \alpha = \frac{1}{5} \quad \beta > \alpha$ ✗ ۹

② $\alpha^2 + \alpha\beta = -\frac{4}{25} \Rightarrow (-\frac{1}{5})^2 + (-\frac{\beta}{5}) = -\frac{4}{25} \Rightarrow \beta = 1 \;, \alpha = -\frac{1}{5} \quad \beta > \alpha$ ✓

۲

$x_v = -\frac{b}{2a} = \frac{2}{5}$

$y_v = -5(\frac{2}{5})^2 + 4(\frac{2}{5}) + 1 \Rightarrow y_v = \frac{9}{5} \qquad V = (\frac{2}{5}, \frac{9}{5}) \to$ ربع اول

$a + b = -\frac{-(a^2 + b^2 - 12)}{1} \Rightarrow a + b = a^2 + b^2 - 12$ ① $\quad a \times b = \frac{a + b - 1}{1} \Rightarrow$ ② $a \times b = a + b - 1 \Rightarrow$

② $a + b = ab + 1 \qquad a^2 + b^2 = (a+b)^2 - 2ab \xrightarrow{①} a + b = [(a+b)^2 - 2ab] - 12 \xrightarrow{③}$ ۱۰

۲

$[ab + 1] = [(ab+1)^2 - 2ab] - 12$ $\qquad$ ③ $S = P + 1 \to P = S - 1$ $\qquad$ ① $S = (a^2 + b^2) - 12 \;, a^2 + b^2 = S^2 - 2P$

$S = (S^2 - 2P) - 12 \xrightarrow{P = S - 1} S = S^2 - 2(S - 1) - 12 \Rightarrow S = S^2 - 2S - 10 = 0 \Rightarrow (S - 5)(S + 2) = 0$

$S = 5$ یا $S = -2 \xrightarrow{\text{طبیعی}} S = a + b \geq 1 + 1 = 2 \quad S = -2$ ✗ $\quad S = 5$ ✓

۴

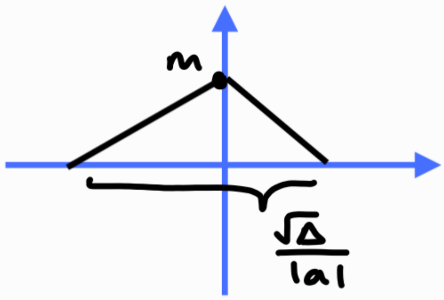
$$\sqrt{\alpha}-\sqrt{\beta}=۱ \xrightarrow{\text{توان ۲}} \alpha+\beta-۲\sqrt{\alpha\beta}=۱ \rightarrow ۳m-۲\sqrt{m}=۱ \quad (\sqrt{m}=t)$$

$$۳t^۲-۲t-۱=۰ \rightarrow \begin{cases} t=۱ \\ t=-\frac{۱}{۳} \end{cases} \xrightarrow{\sqrt{m}>۰} m=۱$$

$$۲x^۲-mx-m=۰ \rightarrow ۲x^۲-x-۱=۰ \rightarrow \boxed{\frac{c}{a}=\frac{-۱}{۲}}$$

۵

$$S=\frac{۱}{\cancel{۲}}\times m\times\frac{\sqrt{m^۲+۴-۴m}}{\cancel{۲}}=\left|\frac{۳}{\cancel{۴}}\right|$$



$$m|m-۲|=|۳| \rightarrow \begin{cases} m|m-۲|=۳ & 1 \\ m|m-۲|=-۳ & 2 \end{cases}$$

1

$$\boxed{m\geq ۲} \rightarrow m^۲-۲m-۳=۰ \rightarrow \begin{cases} \boxed{m=۳} \\ m=-۱ \end{cases}$$

if $m<۲ \rightarrow \Delta<۰$ غ‌ق‌ق

2

$$\boxed{m\leq ۲} \rightarrow -m^۲+۲m+۳=۰ \rightarrow \begin{cases} \boxed{m=-۱} \\ m=۳ \end{cases}$$

if $m>۲ \rightarrow \Delta<۰$ غ‌ق‌ق

$$m=۳ \rightarrow y=x^۲+۳x+۲ \rightarrow \boxed{x_S=-\frac{۳}{۲}}$$

$$m=-۱ \rightarrow y=x^۲-x+۲ \rightarrow \boxed{x_S=-\frac{۱}{۲}}$$