۱) الف) $y = 2x^2 - 4x + 1$

$ext \left| \begin{array}{l} \frac{-b}{2a} = \frac{4}{4} = 1 \\ \frac{-\Delta}{4a} = \frac{-8}{8} = -1 \end{array} \right.$

$\Delta = b^2 - 4ac \Rightarrow 16 - 8 = 8$

ب) $y = -2x^2 + 3x - 5$

$ext \left| \begin{array}{l} \frac{-b}{2a} = \frac{-3}{-4} = \frac{3}{4} \\ \frac{-\Delta}{4a} = \frac{31}{8} \end{array} \right.$

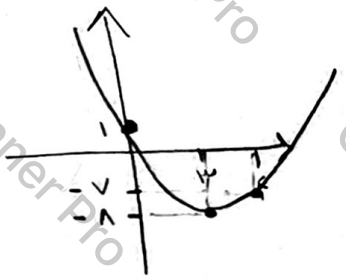
$\Delta = b^2 - 4ac \Rightarrow 9 - 40 = -31$

۲) الف) $y = x^2 - 6x + 1$

$ext \left| \begin{array}{l} \frac{-b}{2a} = \frac{6}{2} = 3 \\ \frac{-\Delta}{4a} = \frac{-32}{4} = -8 \end{array} \right.$

x	4	3	0	1
y	-8	-8	1	-4

$\Delta = b^2 - 4ac \Rightarrow 36 - 4 = 32$

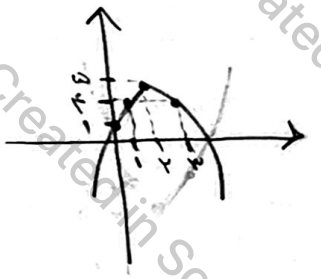


ب) $y = -x^2 + 4x + 1$

$ext \left| \begin{array}{l} \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ \frac{-\Delta}{4a} = \frac{-20}{-4} = 5 \end{array} \right.$

x	1	2	3	0
y	4	5	4	1

$\Delta = b^2 - 4ac \Rightarrow 16 - (-4)(1) = 20$



۳) $4x^3 + kx^2 - 9x - 2 = 0 \longrightarrow a(x-\alpha)(x-\beta)(x-\gamma)$

$= ax^3 - a(\alpha+\beta+\gamma)x^2 + a(\alpha\beta+\alpha\gamma+\beta\gamma) - a\alpha\beta\gamma = 0$

$S = \frac{\alpha+\beta+\gamma}{} = \frac{-k}{4} \rightarrow 1 - \frac{1}{2} = \frac{-k}{4} \rightarrow \boxed{k = -3}$

$P = \frac{\alpha\beta\gamma}{} = \frac{+2}{4} = \frac{+1}{2} \rightarrow \alpha\beta = -2 \quad \gamma = -\frac{1}{4}$

۴) $x^2 - 2mx + m = 0 \xrightarrow{x=\alpha} \alpha^2 - 2m\alpha + m = 0$

$\xrightarrow{x=\beta} \beta^2 - 2m\beta + m = 0$

$S = 2$

$P = m$

$\sqrt{\alpha} - \sqrt{\beta} = 1$ (به توان ۲)

$\alpha + \beta - 2\sqrt{\alpha\beta} = 1$

$S \qquad P$

$2 - 2\sqrt{m} = 1 \rightarrow -2\sqrt{m} = -1 \rightarrow \sqrt{m} = 1 \rightarrow m = +1 \xrightarrow{\sqrt{\ }} m = 1$

$2x^2 - mx - m = 0$

$2x^2 - x - 1 = 0$

$P = \frac{-1}{2}$

$P = \frac{c}{a}$

(ω

$y = 2x^2 - (m-2)x + m$

$S = \frac{1}{2} \times \frac{|m-2|}{2} \times |m| = \frac{|m(m-2)|}{4} = \frac{3}{2}$

$|m(m-2)| = 3 \rightarrow m^2 - 2m - 3 = 0 \rightarrow (m-3)(m+1) = 0$

$m = 3 \quad m = -1$

رأس سهمی $= (0, m) \rightarrow$ روی محور عرض‌ها

$\alpha + \beta = \frac{m+2}{2}$

$\alpha\beta = \frac{m}{2}$

طول پاره‌خط $= |x_1 - x_2| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{(m-2)^2}}{2} = \frac{|m-2|}{2}$

$\Delta = (m+2)^2 - 8m \rightarrow m^2 + 4 + 4m - 8m = m^2 + 4 - 4m = (m-2)^2$

(4

$\frac{m}{2} = \frac{3}{2}, -\frac{1}{2}$

$y = ax^2 + 2x + a$

$\Delta = 4 + 4a^2 = 4(1 + a^2)$

$a > 0$ min $\Delta = 4 + 4a^2$ $a = \frac{1 \pm \sqrt{2}}{1}$ $\rightarrow \begin{cases} a = 1, a = -\frac{1}{1} \cap \rightarrow a = 1 \\ a > 0 \end{cases}$

ext $\left| \begin{array}{l} \frac{-b}{a} = \frac{-2}{2a} = -\frac{1}{a} \\ \frac{-\Delta}{4a} = \frac{4a^2 - 4}{4a} \xrightarrow{b=1} \frac{4a^2 - 4}{4a} = \frac{1}{2} \end{array} \right.$ $\rightarrow 2(4a^2 - 4) = 1a \quad 8a^2 - 8 = 1a$

(1

$x^2 - (a+1)x + a = 0 \xrightarrow{x=\alpha} \alpha^2 - (a+1)\alpha + a = 0 \quad 8a^2 - 1a = 0$

$\frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{(a-1)^2}}{|1|} = a - 1 = 2$ (دوتایی) $\xrightarrow{x=\beta} \beta^2 - (a+1)\beta + a = 0$

$a = 3$ $S = a + 1 = 4$

$P_1 = a = 3$

$\Delta = b^2 - 4ac \rightarrow (a+1)^2 - 4a \rightarrow a^2 + 1 + 2a - 4a = a^2 - 2a + 1$

$(a-1)^2$

$x^2 - (3a+1)x + b = 0 \xrightarrow{x=\alpha} \alpha^2 - (3\alpha+1)\alpha + b = 0 \quad \alpha^2 - 8\alpha + b = 0$

$\xrightarrow{x=\beta} \beta^2 - (3a+1)\beta + b = 0 \quad \beta^2 - 8\beta + b = 0$

$\frac{\sqrt{\Delta}}{|a|} = \sqrt{100 - 4b} = 2$ (دوتایی) $\quad S = 3a + 1 = 10$

$100 - 4b = 4$

$P_2 = b = 24$

$\Delta = b^2 - 4ac \rightarrow (3a+1)^2 - 4b \rightarrow 100 - 4b$

$99 = 4b$

$b = \frac{99}{4} = 24$

اختلاف حاصل‌ضرب ریشه‌ها $= P_2 - P_1 = 24 - 3 = 21$

(8

① $y = -ax^2 + 8x + 2 \rightarrow$ ext $\left| \begin{array}{l} \frac{-b}{2a} = \frac{-8}{-2a} = \frac{1}{2} \\ \frac{-\Delta}{4a} = \frac{a^2 + 8a}{-2a} = \frac{-2a(a+8)}{-4a} = \frac{a+8}{4} \end{array} \right.$ جایگذاری در معادله اول $\begin{cases} a = -12 \end{cases}$

$\Delta = a^2 + 8a$

② $y = 2bx^2 - bx - 1 \rightarrow$ ext $\left| \begin{array}{l} \frac{-b}{2a} = \frac{b}{4b} = \frac{1}{4} \\ \frac{-\Delta}{4a} = \frac{-(b^2 + 8b)}{8b} = \frac{-b(b+8)}{8b} = -\frac{b+8}{8} \end{array} \right.$ جایگذاری در معادله اول $\begin{cases} b = 24 \end{cases}$

$\Delta = b^2 + 8b$

$y = 2\omega\alpha x^2 + 4x + \beta$ $\xrightarrow{x=\alpha}$ $2\omega\alpha^2 + 4\alpha + \beta = 0$

$\xrightarrow{x=\beta}$ $2\omega\alpha\beta^2 + 4\beta + \beta = 0$ $\quad 2\omega\alpha\beta^2 + \omega\beta = 0$

$\omega\beta(\omega\alpha\beta + 1) = 0$

$S = \frac{-4}{2\omega\alpha} = \alpha + \beta$

$P = \frac{\beta}{2\omega\alpha} = \alpha\beta \rightarrow 2\omega\alpha^2 = 1 \rightarrow \alpha = \pm\frac{1}{\omega}$

$\alpha < \beta \xrightarrow{} \alpha = -\frac{1}{\omega}, \beta = 1$

$y = -\omega x^2 + 4x + 1 \rightarrow ext \begin{cases} \frac{2}{\omega} > 0 \\ 1 > 0 \end{cases} \rightarrow$ همه x ها

(10

$x^2 - (a^2+b^2-12)x + a + b - 1 = 0 \quad S = a+b \quad P = ab$

$a^2+b^2-12 = a+b \rightarrow S^2 - 2P - 12 = S \rightarrow S^2 - 2(S-1) - 12 = S$

$ab = a+b-1 \rightarrow P = S-1 \quad S^2 - 2S + 2 - 12 = S$

$S^2 - 3S - 10 = 0$

$(S-5)(S+2) = 0$

$S = +5$ $\quad S = -2$

چون a, b طبیعی‌اند