نام و نام خانوادگی: تانیا ترک ترابی سری: ۲۵ گروه: دهم ریاضی

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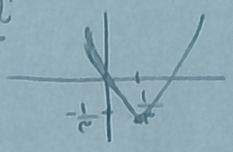
الف) $y = 3x^2 - 2x$ $\min \left| \begin{array}{l} -\frac{b}{2a} = \frac{1}{3} \\ 3 \times \frac{1}{9} - \frac{2}{3} = \frac{1}{3} - \frac{2}{3} = -\frac{1}{3} \end{array} \right.$

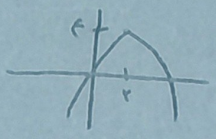
ناصیه‌سم



x	0	$\frac{1}{3}$	$\frac{2}{3}$
y	0	$-\frac{1}{3}$	0

ب) $y = -x^2 + 4x$ $\max \left| \begin{array}{l} -\frac{b}{2a} = 2 \\ -4 + 8 = 4 \end{array} \right.$

ناصیه دوم



x	0	2	4
y	0	4	0

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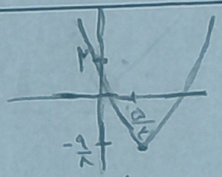
الف) $y = 2x^2 - 5x + 2$ $\min \left| \begin{array}{l} -\frac{b}{2a} = \frac{5}{4} \\ 2 \times \frac{25}{16} - \frac{25}{4} + 2 = \frac{25 - 50 + 16}{8} = -\frac{9}{8} \end{array} \right.$

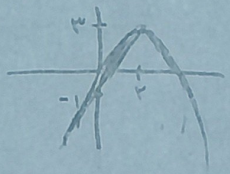
x	0	$\frac{5}{4}$	2
y	2	$-\frac{9}{8}$	2



ناصیه ۱ و ۲

ب) $y = -x^2 + 2x - 1$ $\max \left| \begin{array}{l} -\frac{b}{2a} = 1 \\ -1 + 2 - 1 = 0 \end{array} \right.$

x	0	1	2
y	-1	0	-1



ناصیه ۱ و ۲

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الف) $\frac{\alpha + \beta}{\alpha - \beta} = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$ $\alpha - \beta = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{52}}{1}$ $\alpha + \beta = -\frac{b}{a} = 1$ $\Delta = 1 + 12 \Rightarrow \Delta = 13$

ب) $\alpha^2 + \beta^2 = S^2 - 2P = 1 + 6 = 7$ $S = -\frac{b}{a} = 1$ $P = \frac{c}{a} = -3$

ج) $\alpha^3 + \beta^3 = S^3 - 3SP = 1 + 9 = 10$

د) $\alpha^3 - \beta^3 = \underbrace{(\alpha - \beta)}_{\sqrt{13}}(\underbrace{\alpha^2 + \alpha\beta + \beta^2}_{(\alpha^2 + \beta^2) + \alpha\beta = 7 - 3 = 4}) = \sqrt{13} \times 4 = 4\sqrt{13}$

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$y = \underbrace{(x - 2)}_{x \geq 2}(\underbrace{x^2 - ax + a}_{\Delta < 0})$

چون یک ریشه داریم

$a^2 - 4a < 0 \Rightarrow a(a - 4) < 0$

$a > 0$ و $a < 4$

	0		4	
	$+$	0	$-$	0

$(0, 4)$

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$\Delta = 144 + 12a$ $\alpha, \beta = \frac{12 \pm \sqrt{144 + 12a}}{6}$ $\alpha + \beta = \frac{12}{3} = 4$ $\alpha\beta = \frac{-a}{3}$

$\alpha^2 + \beta^2 = S^2 - 2P = 16 + \frac{2a}{3}$ $\beta^2 = \alpha^2 + \beta^2 - \alpha^2 = (16 + \frac{2a}{3}) - \alpha^2$ $2\alpha^2 + (16 + \frac{2a}{3}) - 4\alpha = 7 \Rightarrow$

$2\alpha^2 - 4\alpha + \frac{2a}{3} + 9 = 0$ $a = \frac{4 \times ((-4)^2 - 4 \times 2 \times 9)}{2} = 2$ $\frac{a}{\alpha} = \frac{2}{2 + \frac{\sqrt{42}}{3}}$

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$A=(2a+3,\ \underset{\text{برابر}}{a-2}),\ B(7-2a,\ \underset{\text{برابر}}{a-2})$ $\qquad y=a(x-5)^2+3 \Rightarrow 1=16a+3$

$x_{\text{راس}}=\dfrac{2x+3+7-2a}{2}=5 \quad \Big|\ \begin{matrix}5\\3\end{matrix}$ $\qquad a=-\dfrac{1}{8}$

$\boxed{b=5} \qquad b-2=3$

$y=0=-\dfrac{1}{8}(x-5)^2+3$

$y(0)=-\dfrac{25}{8}-\dfrac{1}{8} \Rightarrow$ فاصله خواهد شد

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$\begin{cases}\alpha+\beta=1 \rightarrow \alpha=1-\beta\\ \alpha\beta=\dfrac{-b}{a}\end{cases}$

$40\beta^2+20(1-\beta)^2-20\beta=17$

$(1-\beta)^2=1-2\beta+\beta^2$

$40\beta^2+20(1-2\beta+\beta^2)=17 \Rightarrow (40\beta^2+20\beta^2)-(40\beta+20\beta)+20-17=0$

$\Delta=3600-720=2880$

$\beta=\dfrac{1\pm 2\sqrt5}{5}$

$\alpha_1=1-\beta_1,\ \alpha_2=1-\beta_2 \qquad \alpha-\beta=\dfrac{\sqrt{a^2+4ab}}{a}\xrightarrow{\alpha+\beta=1}\dfrac{\sqrt{320}}{20}=\dfrac{8\sqrt5}{20}=\boxed{\dfrac{2\sqrt5}{5}}$

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$(-5,\beta),(1,\beta)\Rightarrow \dfrac{-5+1}{2}=-2 \rightarrow x=-2 \qquad ext\Big|\begin{matrix}-2\\-\frac12\end{matrix} \rightarrow -\dfrac{b}{2a}=-2 \rightarrow b=4a$

$-\dfrac{\Delta}{4a}\Rightarrow \dfrac{-b^2+4ac}{b}=-b+c \ \dfrac{c}{2}=-\dfrac12 \Rightarrow b=2$

$\dfrac12x^2+2x+\dfrac{c}{2}\xrightarrow{x=1}\dfrac12+2+\dfrac{c}{2}\Rightarrow \boxed{b=4}$

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$\alpha+\beta=-6,\ \alpha\beta=a \xrightarrow{\alpha<\beta<0} \beta=-3+d,\ \alpha=-3-d \xrightarrow{d>0} 3\alpha^2+2\beta^2=12\sqrt2+85 \longrightarrow$

$3(-3-d)^2+2(-3+d)^2=12\sqrt2+85 \rightarrow 3(9+6d+d^2)+2(d^2-6d+9)=12\sqrt2,$

$(27+18d+3d^2)+(2d^2-12d+18)=1 \rightarrow 5d^2+6d+45=12\sqrt2+85 \Rightarrow 5d^2+6d-40-12\sqrt2=0$

$\Rightarrow d=\dfrac{-6\pm\sqrt{836+240\sqrt2}}{10} \qquad a=(-3-d)(-3+d)=(-3)^2-d^2=9 \qquad d^2=\dfrac{40+12\sqrt2-6d}{5}$

$a=9-d^2 \Rightarrow a\simeq 9-8=\boxed{1}$

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$\alpha+\beta=\dfrac{m+14}{36},\ \alpha\beta=\dfrac{1}{36}$

$\sqrt{\dfrac1\alpha}+\sqrt{\dfrac1\beta}=5=\dfrac{\sqrt\beta+\sqrt\alpha}{\sqrt{\alpha\beta}}=5 \Rightarrow \sqrt\alpha+\sqrt\beta=5\times\dfrac16=\dfrac56 \qquad \left(\sqrt{\alpha\beta}=\sqrt{\dfrac{1}{36}}\right)$

$\alpha+\beta=\left(\dfrac56\right)^2=\dfrac{25}{36} \qquad \dfrac{m+14}{36}=\dfrac{25}{36}\Rightarrow m=11$

حاصل ضرب ریشه های معادله دوم

$\dfrac{2}{m}=\boxed{\dfrac{2}{11}}$