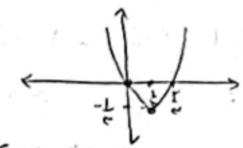
دهم دفتر B — سحر میرزاده — تست شماره‌ی ۲۵

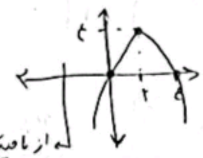
۱)

الف) $y = 3x^2 - 2x \rightarrow x(3x-2)$ $S \left| \begin{matrix} -\frac{b}{2a} \\ -\frac{\Delta}{4a} \end{matrix} \right. \rightarrow S \left| \begin{matrix} \frac{1}{3} \\ -\frac{1}{3} \end{matrix} \right.$ $A \left| \begin{matrix} 0 \\ 0 \end{matrix} \right.$ (مبدأ) $B \left| \begin{matrix} \frac{2}{3} \\ 0 \end{matrix} \right.$



← از ناحیه‌ی سوم نمی‌گذرد

ب) $y = -x^2 + 4x \rightarrow -x(x-4)$ $S \left| \begin{matrix} 2 \\ 4 \end{matrix} \right.$ $A \left| \begin{matrix} 0 \\ 0 \end{matrix} \right.$ $B \left| \begin{matrix} 4 \\ 0 \end{matrix} \right.$



← از ناحیه‌ی دوم نمی‌گذرد

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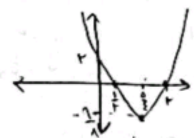
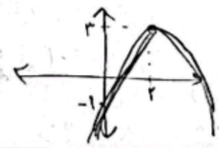
۲) الف) $y = 2x^2 - 5x + 2$ $\frac{25}{16}\cdot 2 - \frac{25}{4} + 2 = \frac{25-50+16}{8} = -\frac{9}{8}$ $S \left| \begin{matrix} \frac{5}{4} \\ -\frac{9}{8} \end{matrix} \right.$ $A \left| \begin{matrix} \frac{1}{2} \\ 0 \end{matrix} \right.$ $B \left| \begin{matrix} 2 \\ 0 \end{matrix} \right.$

$(2x-1)(x-2)$



از نواحی ۲ و ۴ می‌گذرد

ب) $y = -x^2 + 4x - 1$ $S \left| \begin{matrix} 2 \\ 3 \end{matrix} \right.$ $C \left| \begin{matrix} 0 \\ -1 \end{matrix} \right.$



از نواحی ۳ و ۴ می‌گذرد

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۳) $x^2 - x - 3 = 0$ $\alpha + \beta = \frac{+1}{1} = +1$, $\alpha\beta = -3$, $\alpha - \beta = \frac{\sqrt{1+12}}{1} = \sqrt{13}$

الف) $\frac{\alpha+\beta}{\alpha-\beta} = \frac{+1}{\sqrt{13}} = \frac{+\sqrt{13}}{13}$ ب) $\alpha^2 + \beta^2 = S^2 - 2P = 1 + 6 = 7$ ج) $\alpha^3 + \beta^3 = S^3 - 3SP = 1 + 9 = 10$

د) $\alpha^3 - \beta^3 = (\alpha-\beta)(\alpha^2 + \beta^2 + \alpha\beta) = \sqrt{13}\,(\underbrace{7-3}_{4}) = 4\sqrt{13}$

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۴) $y = (x-2)(x^2 - ax + a) \rightarrow$ نقطه‌ی مشترک $\Longrightarrow y = x^2 - ax + a$

ریشه نباید داشته باشد، یا ریشه‌ی مضاعف

$\Delta < 0$ $a^2 - 4a < 0$ $a(a-4) < 0$ (جدول علامت: ۰ ، ۴ : + | − | +)

$\Longrightarrow a \in (0, 4) \Rightarrow \boxed{0 < a \leq 4}$ اگر ریشه ۲ $\rightarrow (x-2)^2 \Rightarrow a = 4$

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۵) $3x^2 - 12x - a = 0$ ریشه‌ها α, β , $2\alpha^2 + \beta^2 - 4\alpha = 7$, $\alpha + \beta = 4$ $\alpha\beta = -\frac{a}{3}$

$\Rightarrow \alpha^2 + \beta^2 + \underbrace{\alpha^2 - 4\alpha}_{} = 7$
$3\alpha^2 - 12\alpha = a \rightarrow \alpha^2 - 4\alpha = \frac{a}{3}$
$\Big\} \Rightarrow 16 + \frac{2a}{3} + \frac{a}{3} = 7 \rightarrow 16 + a = 7 \Rightarrow a = -9 \Rightarrow \alpha\beta = 3$

$\Rightarrow 3x^2 - 12x + 9 = 0 \xrightarrow{\div 3} x^2 - 4x + 3 = 0 \Rightarrow (x-1)(x-3) = 0$ $x = 1$ ، $x = 3$ ریشه‌ی بزرگتر $\rightarrow \frac{a}{3} = \frac{-9}{3} = \boxed{-3}$

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۶) $A(2a+3, a-2)$, $B(7-2a, a-2)$ عرض یکسان → میانگین طول‌ها $= x_S$ $S(b, b-2)$

$b = \frac{2a+3+7-2a}{2} = 5 \rightarrow S(5, 3)$ $y = d(x-5)^2 + 3 \leftarrow$ $1 = d(-4)^2 + 3 \rightarrow d = -\frac{1}{8}$

$y = -\frac{1}{8}x^2 + \frac{10}{8}x - \frac{1}{8} \rightarrow c = -\frac{1}{8}$ $\boxed{-\frac{1}{8}}$ $A(9,1)$, $B(1,1)$

دو لغزها مثبت $\Rightarrow 2a+3 > 0 \rightarrow 2a > -3 \rightarrow a > -\frac{3}{2}$

$7 - 2a > 0 \rightarrow a < 3/5$

$a - 2 > 0 \rightarrow a > 2$ $\Big\}$ $a \in (2, 3/5)$ چون عدد صحیح است $a = 3$

$\Rightarrow$ $\boxed{\text{جواب} = \frac{1}{8}}$

۸)

$ax^2 - ax - b = 0$ ، α, β ریشه‌ها ، $20\beta^2 + 20\alpha^2 - 20\beta = 17$

$\alpha + \beta = \frac{a}{a} = 1$ $\Rightarrow 20\alpha^2 + 20\beta^2 + 20\beta^2 - 20\beta = 17 \Rightarrow 20(\alpha^2+\beta^2) + 20\beta(\underbrace{\beta-1}_{-\alpha}) = 17$

$20(1 - 2\alpha\beta) - 20\alpha\beta = 20 - 40\alpha\beta - 20\alpha\beta = 17 \Rightarrow 20 - 60\alpha\beta = 17 \Rightarrow 60\alpha\beta = 3$

$\Rightarrow \alpha\beta = \frac{1}{20} = \frac{-b}{20a} \Rightarrow a = -20b$ $\quad -20bx^2 + 20bx - b$

$\alpha - \beta = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{400b^2 - 320b^2}}{|-20b|} = \frac{\sqrt{80b^2}}{20|b|} = \frac{4\sqrt{5}|b|}{20|b|} = \boxed{\frac{\sqrt{5}}{5}}$

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۹)

$(-3, \beta)$ ، $(1, \beta)$ $\quad x_S = \frac{1-3}{2} = -1 \Rightarrow S \mid \begin{matrix} -1 \\ -\frac{1}{4} \end{matrix}$ $\quad \Rightarrow y = ax^2 + bx + c \Rightarrow \frac{-b}{2a} = -1 \Rightarrow b = 2a$

$y_S = -\frac{1}{4}$

$C(0, \frac{3}{4})$ $\xrightarrow[y=-\frac{1}{4}]{x=-1}$ $4a - 2b + \frac{3}{4} = -\frac{1}{4} \Rightarrow 4a - 2b = -1 \Rightarrow 4a - 4a = -1 \Rightarrow 0 = -1$ ✗

$a = \frac{1}{4}$ ✓

$\Rightarrow y = \frac{1}{4}x^2 + 2x + \frac{3}{4} \Rightarrow \beta = \frac{1}{4} + 2 + \frac{3}{4} = \boxed{3}$

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۱۰)

$\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 3$ $\quad$ $4x^2 - (m+13)x + 1 = 0$

$\alpha + \beta = \frac{m+13}{4}$ $\quad \alpha\beta = \frac{1}{4}$

$\frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}} = 3 \rightarrow \frac{\alpha+\beta+2\sqrt{\alpha\beta}}{\alpha\beta} = 9 \Rightarrow \frac{\frac{m+13}{4} + \frac{1}{2}}{\frac{1}{4}} = \frac{\frac{m+15}{4}}{\frac{1}{4}} = 9 \Rightarrow m + 15 = 9$

$\Rightarrow m = -1$

$\rightarrow mx^2 + 2x + 2 = -x^2 + 2x + 2 = 0 \longrightarrow$ حاصل ضرب ریشه‌ها $= -2$