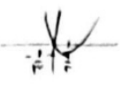
(۱)

بیان اشکال دارد (مبهم است)

$y = 3x^2 - 2x \rightarrow$ مینیمم دارد

$a>0$ Min

راس $\left|\begin{array}{l} -\frac{b}{2a} = \frac{2}{6} = \frac{1}{3} \\ 3\times\frac{1}{9} - \frac{2}{3} = -\frac{1}{3}\end{array}\right.$



از ناحیه ۳ نمی‌گذرد

$y = -x^2 + 4x \rightarrow$ ماکزیمم دارد

$a<0$ Max

راس $\left|\begin{array}{l} -\frac{b}{2a} = \frac{-4}{-2} = 2 \\ -(2)^2 + 4(2) = 4\end{array}\right.$

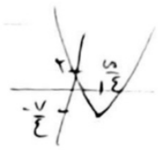
از ناحیه ۲ نمی‌گذرد

(۲)

Min

$y = 2x^2 - 5x + 2$

$\left|\begin{array}{l} -\frac{b}{2a} = \frac{5}{4} \\ 2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) + \frac{8}{4} = \frac{25 - 50 + 16}{8} = -\frac{14}{8} = -\frac{7}{4}\end{array}\right.$



از ۱ و ۳ و ۴ می‌گذرد

Max

$y = -x^2 + 4x - 1$

$\left|\begin{array}{l} -\frac{b}{2a} = \frac{-4}{-2} = 2 \\ -(4) + 4(2) - 1 = 3\end{array}\right.$

از ۱ و ۳ و ۴ می‌گذرد

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(۳)

$x^2 - x - 3 = 0$, $S = 1$, $P = -3$

الف) $\frac{\alpha+\beta}{\alpha-\beta} = \frac{1}{\frac{\sqrt{\Delta}}{|a|}} = \frac{1}{\sqrt{1-4(-3)}} = \frac{1}{\sqrt{13}}$ ($\Delta = b^2 - 4ac$)

ب) $\alpha^2 + \beta^2 = S^2 - 2P \rightarrow 1^2 - 2(-3) = 7$

ج) $\alpha^3 + \beta^3 = S^3 - 3SP \rightarrow 1^3 - 3(1)(-3) = 10$

د) $\alpha^3 - \beta^3 = (\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta) = (\sqrt{13})(7-3) = 4\sqrt{13}$

$\frac{\sqrt{\Delta}}{|a|} = \sqrt{13}$, 7, $P = -3$, (۴)

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(۴)

$y = (x-2)(x^2 - ax + a)$

$x = 2$, $\Delta < 0$

$b^2 - 4ac < 0 \quad a^2 - 4a < 0$

$a(a-4) < 0$

$+ \;|\; - \;|\; +$ (۰ و ۴)

$a \in (0, 4)$

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(۵)

$2\alpha^2 + \beta^2 - 4\alpha = 7 \rightarrow \alpha^2 + \beta^2 + \underbrace{\alpha^2 - 4\alpha}_{\frac{a}{3}} = 7 \rightarrow 16 + \frac{2}{3}a + \frac{a}{3} = 7$

$a = -9$

$3\alpha^2 - 12\alpha - a = 0 \rightarrow \alpha^2 - 4\alpha = \frac{a}{3}$ $\quad 3\beta^2 - 12\beta - a = 0$

$3x^2 - 12x + 9 = 0 \rightarrow 3(x^2 - 4x + 3) = 0 \rightarrow \frac{4 \pm \sqrt{4}}{2}$ (3, 1)

$-\frac{9}{3} = (3)$

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(۶)

$B, A \rightarrow$ عرض برابر $\xrightarrow{\text{پس}} \frac{7 - 2a + 2a + 3}{2} = 5 \rightarrow b =$ طول راس $\rightarrow$ راس $\left|\begin{array}{l}5\\3\end{array}\right.$ $\quad ax^2 + bx + c = 0$

برای طبیعی شدن مولفه‌ها باید $a = 3$ باشد $\xleftarrow{\text{پس}}$ $A(9,1), B(1,1)$

$25a + 5b + c = 3 \rightarrow 25a - 50a + c = 3 \xleftarrow{\text{تفاضل}}$

$B \rightarrow a + b + c = 1 \rightarrow a - 10a + c = 1$

$\rightarrow -16a = 2$

$a = -\frac{1}{8} \xrightarrow{\text{جایگذاری}} c = -\frac{1}{8} \xrightarrow{\text{فاصله}} \left(\frac{1}{8}\right)$

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(۷)

$ax^2 - ax - b = 0 \quad 40\beta^2 + 20\alpha^2 - 20\beta = 17 \rightarrow 40\beta^2 + 20(1-\beta)^2 - 20\beta = 17 \rightarrow 60\beta^2 - 60\beta + 3 = 0$

$\leftarrow 20\beta^2 - 20\beta + 1 = 0$

$\beta = \frac{20 \pm 8\sqrt{5}}{40}$ ($\frac{5+2\sqrt{5}}{10}$, $\frac{5-2\sqrt{5}}{10}$) $\xrightarrow{\text{پس}}$ $\alpha = \frac{5-2\sqrt{5}}{10}$, $\alpha = \frac{5+2\sqrt{5}}{10}$

$\rightarrow |\alpha-\beta| \rightarrow \left|-\frac{2\sqrt{5}}{5}\right| = \left(\frac{2\sqrt{5}}{5}\right)$

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(۸)

$(-5, \beta), (1, \beta) \xrightarrow{\text{پس}}$ طول راس $= -2 \longrightarrow$ راس $= \left|\begin{array}{l}-2\\-\frac{1}{2}\end{array}\right. \rightarrow y = a(x+2)^2 - \frac{1}{2} \xrightarrow{\left|\begin{array}{l}0\\ \frac{3}{2}\end{array}\right.}$

عرض از مبدا $\Rightarrow \left|\begin{array}{l}0\\ \frac{3}{2}\end{array}\right.$ $\quad y = \frac{1}{2}(x+2)^2 - \frac{1}{2} \leftarrow a = \frac{1}{2} \leftarrow 2 = \frac{4}{2} = 4a \leftarrow \frac{3}{2} = 4a - \frac{1}{2}$

$\xrightarrow{x=-5} \frac{1}{2}(-3)^2 - \frac{1}{2} \rightarrow 4 = \beta$

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(۹)

$\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 5 \xrightarrow{\text{توان ۲}} \overbrace{\frac{1}{\alpha} + \frac{1}{\beta}}^{\frac{\alpha+\beta}{\alpha\beta}} + 2\sqrt{\frac{1}{\alpha\beta}} = 25$

$\alpha\beta = \frac{1}{36} \rightarrow \frac{\alpha+\beta}{\frac{1}{36}} + 2\sqrt{36} = 25 \rightarrow 36(\alpha+\beta) = 13$

$-x^2 + 3x + 2 = 0$ $\quad x = -1 \leftarrow \frac{x+14}{36} = \frac{13}{36} \leftarrow \alpha+\beta = \frac{13}{36}$

$\frac{c}{a} = (-2)$ ✓

$\alpha + \beta = -6 \longrightarrow$ طول راس $=$ میانگین دو ریشه $\rightarrow -3 \xrightarrow{\text{طول دو ریشه}} (-3+Z), (-3-Z)$

$\xrightarrow{\text{جایگذاری}} 3(-3+Z)^2 + 2(-3-Z)^2 = 12\sqrt{2} + 85 \rightarrow 5Z^2 + 45 - 6Z = 12\sqrt{2} + 85$

$\rightarrow y = -2\sqrt{2}$ $\quad \alpha = -3 - 2\sqrt{2}$, $\beta = -3 + 2\sqrt{2}$ $\Big) \times \rightarrow 9 - 8 = 1 = a$