۱- $x^2 - ax + b = 0$

$\alpha\beta = 2^3 = b$, $\alpha + \beta = 2^2 = a$, $a + b = 7$ ✓ — گزینه ۲

۲- $y = (x+1)^2 (x-2)^3 (-x+m-1)$

$y' = (x+1)(x-2)^2 (-x+m-1) + (x+1)^3 \times 3(x-2)^2 (-1)$

$= (x+1)(x-2)^2 (-2x+m-4)$

$x = -1$, $x = 2$, $x = \frac{4-m}{2}$

$\frac{4-m}{2} = 2 \Rightarrow m = 0$

$\frac{4-m}{2} = -1 \Rightarrow m = 6$ ✓

$x = 1 \Rightarrow y > 0 \Rightarrow (-1+m-2)(4) > 0$

$m - 3 > 0 \Rightarrow m > 3$ ✓

$m = 6 \Rightarrow 4 > 3$ ✓ — گزینه ۲

۳- $y = -\frac{1}{2}x^2 + 2x + 6$

$-\frac{1}{2}x^2 + 2x + 6 > \frac{7}{2}$

$-\frac{1}{2}x^2 + 2x + \frac{5}{2} > 0$

$x^2 - 4x - 5 < 0 \Rightarrow (x-5)(x+1) < 0$

$-1 < x < 5 \Rightarrow (a,b) = (-1,5) \Rightarrow b - a = 6$ ✓ — گزینه ۲

۴- $y = x^4 - 2x^3 - x + 2$

$x = 0$

$x^3 - 2x^2 - x + 2 > 0$

$x^2(x-2) - (x-2) > 0$

$(x-2)(x^2-1) > 0$

$x > 2$ ✓ — گزینه ۳

$(a,b) = (2,3) \Rightarrow \frac{b}{a-1} = 2$

$f(2) = 8 - 12 - 2 + 2 = -4$ ✓

۵- $(a-1)x^2 + (a-1)x + 1 > 0$

① $\Delta < 0 \Rightarrow a^2 - 2a - 3 < 0$

② $a - 1 > 0$

$a \in (-1,3)$ ✓

$a^2 - 2a + 3 > 0$ ✓ — گزینه ۲

۶- $\frac{m(m^2+m)}{m-2} = \frac{m^2(m^2+1)}{m-2} > 0$

$m - 2 > 0$

$m > 2$ ✓ — گزینه ۲

۷- $\frac{(x-1)^2(x^2-x-6)}{(x^2+x+1)(x-2)^2} \geq 0$

$\frac{(x-1)^2(x+2)(x-3)}{(x^2+x+1)(x-2)^2} \geq 0$

$x \in (-\infty, -2] \cup [3, +\infty)$ ✓ — گزینه ۲

۸- $\frac{x^2 - 2x - 3}{x^2 + 2} < 2 \Rightarrow x^2 - 2x - 3 < 2x^2 + 4$

$x^2 + 2x + 7 > 0$ ✓

$(3,4) \rightarrow (a,b) = (2,4) \rightarrow b - a = 2$ ✓ — گزینه ۲

۹-

$$-1 < \underbrace{\frac{3x^2 - 4x}{x+1}}_{①} < 0 \quad \underbrace{\qquad}_{②}$$

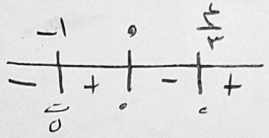
① $\frac{x(3x-4)}{x+1} > -1 \rightarrow \frac{x(3x-4)}{x+1} + 1 > 0 \rightarrow \frac{3x^2 - x + 1}{x+1} > 0$ (−1)

$\Delta = 1 - 12 < 0 \rightarrow$ همیشه + $\quad a > 0$

$x + 1 > 0$

$x > -1$

② $\frac{3x^2 - 4x}{x+1} < 0 \rightarrow \frac{x(3x-4)}{x+1} < 0 \rightarrow$

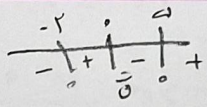


$\rightarrow (-\infty, -1) \cup (0, \frac{4}{3})$

① ∪ ② $\rightarrow (0, \frac{4}{3})$

۱۰-

$\frac{x^2 - 10}{x} \cdot 3 \neq 0 \longrightarrow \frac{x^2 - 3x - 10}{x} \neq 0 \quad \frac{(x-5)(x+2)}{x} \neq 0$



$(-\infty, -2] \cup [0, 5)$