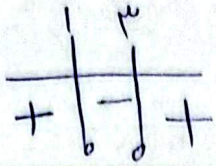
(۱)

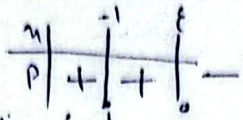
x^2-ax+b



$\begin{cases} -\frac{a}{2}=1+3=4 \\ \frac{b}{2}=3 \end{cases} \Rightarrow a+b=\boxed{1}$ ✓

(۲)

$y=((k-2)x+m-1)(x-4n)^2$



$k \in \mathbb{N}$ — $(x-4n)^2$ ریشه‌ی مضاعف = ریشه‌ی عبارت — عبارت درجه‌ی اول = عبارت خطی

$4n=-1 \Rightarrow \boxed{n=-\frac{1}{4}} \Rightarrow (x+1)^2$

چون $(x+1)^2$ عاده ست ⟸ چون معادله تنها یک جواب دارد ⟸ تنها حالت عبارت اول

در عبارت درجه اول ضریب x برابر صفر است ⟸ $k-2=0 \Rightarrow k=2$ ⟵ چون عضو $\mathbb{N}$ است ⟵ $\boxed{k=1}$

$x=4 \Rightarrow$ عبارت اول صفر می‌شود $\Rightarrow 4k-8+m-1=0 \quad 4k+m-9=0 \Rightarrow m=9-4k \xrightarrow{k=1} \boxed{m=5}$

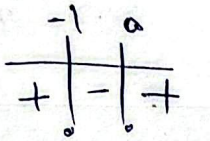
$\frac{m}{n}+k=\frac{5}{-\frac{1}{2}}+1=-10+1=\boxed{-9}$ ✓

(۳)

$y=-\frac{1}{4}x^2+2x+4$

$-\frac{1}{4}x^2+2x+4>\frac{5}{4} \xrightarrow{\times 4} -x^2+8x+16>5 \Rightarrow x^2-8x-11<0$

$(x-5)(x+1)<0$

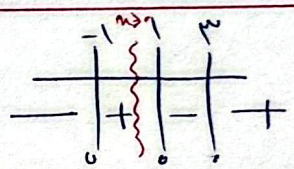


$\Rightarrow (a,b)=(-1,5) \longrightarrow b-a=5+1=\boxed{6}$ ✓

(۴)

$f(x)=x^3-3x^2-x+3$ ⟵ تجزیه ضرایب ثابت $\quad f(x)=(x-1)(x+1)(x-3)$

$x>0$



پایین کوزه‌ها $\Rightarrow y<0 \longrightarrow (a,b)=(1,3)$ ⟵ تقعر یابی $=2 \Rightarrow x=2 \Rightarrow f(x)=(1)(2)(-1)=\boxed{-3}$ ✓

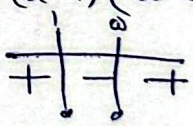
(۵)

$y=(a-1)x^2+(a-1)x+1$ ⟵ همواره مثبت

① $a-1>0 \rightarrow a>1$

$\Delta<0 \rightarrow (a-1)^2-4(a-1)<0 \Rightarrow (a-1)(a-5)<0$

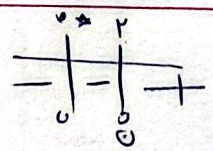
② $a \in (1,5)$



① ∩ ② $=\varnothing$ ✓ ⟵ به ازای هیچ a ای تعلق ندارد

(۶)

$\frac{m(m^3+m)}{m-2}$ ⟵ همواره مثبت $\quad \frac{m^2(m^2+1)}{m-2}$ ⟵ همواره مثبت

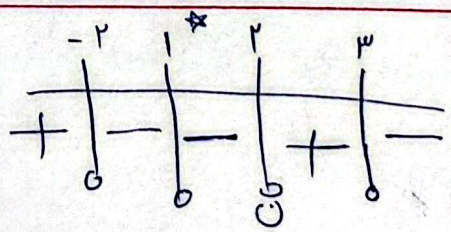


$\Rightarrow m>2$ ✓

(۷)

$\frac{(x^2-x-2)(x-1)^2}{(x^2+x+1)(2-x)^3}<0 \Rightarrow \frac{(x-2)(x+1)(x-1)^2}{(x^2+x+1)(2-x)^3}<0$

⟵ همواره مثبت

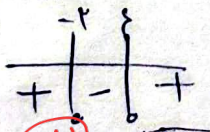


$D_f=[-1,2) \cup [3,+\infty)$ ✓

(۸)

$f(x)=\frac{3x^2-2x}{x^2+4}=\frac{x(3x-2)}{x^2+4} \Rightarrow \frac{3x^2-2x}{x^2+4}<2 \quad 3x^2-2x<2x^2+8$

$x^2-2x-8<0 \quad (x-4)(x+2)<0$



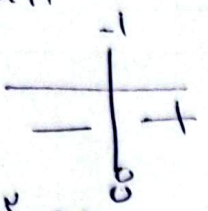
$\Rightarrow (a,b)=(-2,4)$ ✓

$b-a=\boxed{6}$

$-1 < \frac{3x^2-4x}{x+1} < 0$ (۴)

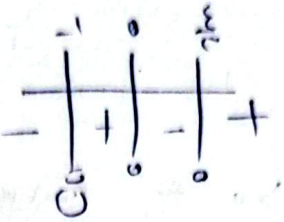
① $\frac{3x^2-4x}{x+1} > -1 \quad \frac{3x^2-4x}{x+1} + 1 > 0 \Rightarrow \frac{3x^2-4x+x+1}{x+1} > 0 \Rightarrow \frac{3x^2-3x+1}{x+1} > 0$

$a > 0$, $\Delta = 9 - 12 = -3 < 0$ → همواره مثبت است



$\Rightarrow x > -1$

② $\frac{3x^2-4x}{x+1} < 0 \quad \frac{x(3x-4)}{x+1} < 0$



$\Rightarrow x < -1$ یا $0 < x < \frac{4}{3}$

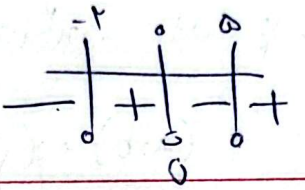
$0 < x < \frac{4}{3}$ ⇓ $x \in (0, \frac{4}{3})$

$J = (-1, \frac{4}{3})$ ②

(10

$\frac{x^2-10}{x} \geq 3 \quad \frac{x^2-10}{x} - 3 \geq 0 \quad \frac{x^2-3x-10}{x} \geq 0$

$\frac{(x-5)(x+2)}{x} \geq 0$



$\Rightarrow x \in (-\infty, -2] \cup (0, 5]$ ②