مهتا شکور — «تابع ها شماره ۲۷» — دهم رشته B

-۱

الف) $\begin{cases} 3x - y = 9 \\ x + 2y = -4 \end{cases} \xrightarrow{\times 2} \begin{cases} 6x - \cancel{2y} = 18 \\ x + \cancel{2y} = -4 \end{cases} \rightarrow 7x = 14$, $\boxed{x = 2} \Rightarrow \boxed{y = -3} \Rightarrow \frac{x}{y} = \boxed{\frac{-2}{3}}$

ب) $\frac{1}{x} - \frac{1}{y} = -1 \rightarrow y - x = -xy$ / $5y - 7x = -3xy \Rightarrow \begin{cases} y - x = -xy \xrightarrow{\times -5} \\ 5y - 7x = -3xy \end{cases} \begin{cases} -5y + 5x = 5xy \\ 5y - 7x = -3xy \end{cases}$

$\Rightarrow -2\cancel{x} = 2\cancel{x}y \Rightarrow y = -1 \Rightarrow -1 - x = x \Rightarrow x = \frac{-1}{2} \Rightarrow \boxed{\frac{x}{y} = \frac{1}{2}}$

-۲

$\underbrace{f(a)}_{2a} + \underbrace{2f(2)}_{2b} = \underbrace{3f(1)}_{-6} \Rightarrow 2a + 2b = -6$, $a + b = -3$ / $(\lg a + 1) = (\lg -2)$, $a + 1 = -2 \Rightarrow a = -3 \Rightarrow \boxed{b = 0}$

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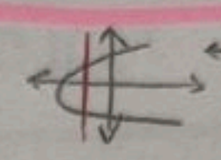
$(-1, m^2 - 3m) = (-1, -2) \Rightarrow m^2 - 3m + 2 = 0 \rightarrow (m-1)(m-2) = 0 \rightarrow m \rightarrow 2, 1$

$(m+1, 6) \rightarrow m = 2 \rightsquigarrow (3, 6) \neq (3, 5)$, $\rightarrow m = 1 \rightsquigarrow (2, 6) \neq (2, 4)$ / $(m^2 + 2, 4m + 1) \rightarrow m = 2 \rightsquigarrow (4, 9)$, $\rightarrow m = 1 \rightsquigarrow (3, 5)$ قابق $\Rightarrow$ هیچ مقدار m

-۴

الف) تابع نیست ← 

ب) تابع هست. ج) تابع نیست. د) تابع هست.

-۵ ×

الف) $y = -\sqrt{x+1} \rightsquigarrow \begin{cases} y_1 = -\sqrt{x+1} \\ y_2 = -\sqrt{x+1} \end{cases} \Rightarrow y_1 = y_2$ تابع هست

ب) × $x = 1 \rightsquigarrow \sqrt{1 - y^2} = y \rightsquigarrow 1 - y^2 = y^2 \rightsquigarrow 2y^2 = 1 \Rightarrow y^2 = \frac{1}{2} \Rightarrow \boxed{y = \pm\frac{1}{2}} \rightarrow$ تابع نیست

-۶ ×

الف) $\begin{matrix} |y_1| = x \\ |y_2| = x \end{matrix} \Rightarrow |y_1| = |y_2| \Rightarrow y_1 = \pm y_2 \rightsquigarrow$ تابع نیست. $\xrightarrow{x=2} |y| = 2 \Rightarrow y = \pm 2 \rightarrow$ غ

ب) × $(y+1)^3 = -x^3 - x + 1 \rightsquigarrow y = \sqrt[3]{-x^3 - x + 1} - 1 \Rightarrow \begin{cases} y_1 = \sqrt[3]{-x^3 - x + 1} - 1 \\ y_2 = \sqrt[3]{-x^3 - x + 1} - 1 \end{cases} \Rightarrow y_1 = y_2$ تابع هست.

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$f(x) = \frac{x^2 + 4x + 5}{x^2 + 4x + 7} \Rightarrow f(x) = \frac{(x^2 + 4x + 4) + 1}{(x^2 + 4x + 4) + 3} = \frac{(x+2)^2 + 1}{(x+2)^2 + 3} \Rightarrow x = \sqrt{3} - 2$

$\frac{(\sqrt{3} \cancel{-2} \cancel{+2})^2 + 1}{(\sqrt{3} \cancel{-2} \cancel{+2})^2 + 3} = \frac{3 + 1}{3 + 3} = \frac{4}{6} = \boxed{\frac{2}{3} = f(\sqrt{3} - 2)}$

-۸ ×

$f(-1) = -4 \Rightarrow -1 - a + b = -4 \Rightarrow b - a = -3$

$y = 3x - a \xrightarrow{(-1, -4)} -4 = -3 - a \Rightarrow a = 1$

$\Rightarrow b = -2 \Rightarrow f(x) = x^3 + x - 2$, $y = 3x - 1$

$x^3 + x - 2 = 3x - 1 \rightsquigarrow x^3 - 2x - 1 = 0 \rightsquigarrow (x+1)(x^2 - x - 1) = 0 \rightsquigarrow x \rightarrow -1, \frac{1 \pm \sqrt{5}}{2}$

$\frac{1 \pm \sqrt{5}}{2}$ ۲ نقطه ۱ دیگه $\rightsquigarrow \frac{1 + \sqrt{5}}{2} + \frac{1 - \sqrt{5}}{2} = 1$

-۹

$a + b = 2a \Rightarrow a = b$

$a - 2b + 1 = 2a$

$\cancel{a} - \cancel{2a} + 1 = 2a \Rightarrow \boxed{a = \frac{1}{3}} \Rightarrow \boxed{b = \frac{1}{3}}$

-۱۰

تابع همانی $\Rightarrow f(x) = x \Rightarrow x = \frac{4x^2 - ax + c + 1}{bx + 3}$

$bx^2 + 3x = 4x^2 - ax + c + 1 \Rightarrow \begin{cases} 3x = -ax \Rightarrow a = -3 \\ bx^2 = 4x^2 \Rightarrow b = 4 \\ c + 1 = 0 \Rightarrow c = -1 \end{cases} \Rightarrow \boxed{a + b + c = 0}$