① الف) $(-4, 3x-y)(9, x+2y)$

$3x-y=9 \xrightarrow{\times 2} 6x-2y=18$

$x+2y=-4$

$7x=14$

$x=2 \Rightarrow y=-3$

$\frac{x}{y}=-\frac{2}{3}$

ب) $(-3,-1)\left(\frac{1}{x}-\frac{3}{y}, \frac{2}{y}-\frac{1}{x}\right)$

$\frac{1}{x}-\frac{2}{y}=-1 \xrightarrow{\times -3} -\frac{3}{x}+\frac{3}{y}=3$

$\frac{2}{y}-\frac{3}{x}=-3$

$\frac{-2}{y}=2 \rightarrow y=-1$

$\frac{1}{x}=\frac{2}{y}-1=-2 \rightarrow x=-\frac{1}{2}$

$\frac{x}{y}=\frac{\frac{1}{2}}{-1}=-\frac{1}{2}$

$\frac{3}{x}+7=-3$

$\frac{3}{x}=-10$

② $a=-2$

$a+1=-2$

$f=\{(a,2a),(1,a+1),(1,-2),(2,b)\}$ $\quad -3,-6$

$f(a)+2f(2)=3f(1)$

$f(-2)+2f(2)=3f(1)$

$-6+2b=-6 \rightarrow b=0$

③ تابع نیست

$f=\{(-1,m^2-3m),(3,3),(-2,-1),(m+1,6),(4,2),(m^2+2,4m+1)\}$

$6,6 \quad 2,6$

$m^2-3m+2=0$

$(m-2)(m-1)=0$

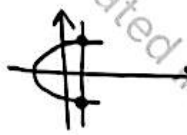
$m=1 \Rightarrow x$ $\quad m=2 \Rightarrow x$

$3 \quad 6$

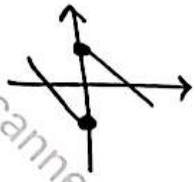
تابع نیست

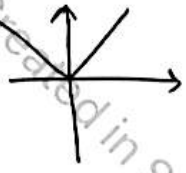
به ازای هیچ مقدار

④

الف)  تابع نیست

ب)  تابع هست

ج)  تابع نیست

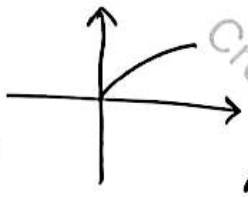
د)  تابع هست

③

الف) $y = -\sqrt{x+1} \Rightarrow y_1 = -\sqrt{x_1+1}$ توان ۲ $\rightarrow y_1^2 = +|x_1+1|$

$y_2 = -\sqrt{x_2+1} \rightarrow y_2^2 = +|x_2+1|$

$x_1 = x_2 \rightarrow y_2^2 = y_1^2 \rightarrow y_1 = \pm y_2$

$\sqrt{x}$ → 

$-\sqrt{x+1}$ → 

تابع هست

ب) $x = \frac{y}{\sqrt{1-y^2}}$ $x=1 \rightarrow y = \sqrt{1-y^2}$ توان ۲ $\rightarrow y^2 = 1-y^2 \rightarrow 2y^2 = 1$

$y^2 = \frac{1}{2}$

$y = \pm\frac{1}{\sqrt{2}}$

تابع نیست

④

الف) $|y| = x \rightarrow \begin{cases} |y_1| = x_1 \\ |y_2| = x_2 \end{cases}$ $x_1 = x_2 \rightarrow |y_1| = |y_2|$

$y_1 = \pm y_2$

تابع نیست

ب) $y^3 + 3y^2 + 3y + x^3 + x = 0$

$x=0 \rightarrow y^3 + 3y^2 + 3y = 0$

$y(y^2 + 3y + 3) = 0$

$y = 0$

تابع هست

تابع $y^3 + y$ → مقدار $y^3 + y = -x^3 - x - 3y^2$

(۷)

$$f(x)=\frac{x^2+4x+3}{x^2+4x+7}=\frac{(x+2)^2}{x^2+4x+4+3}=\frac{(x+2)^2}{(x+2)^2}$$

$$f(\sqrt{x}-2)=\frac{x+1}{x+4}=\frac{4}{6}=\boxed{\frac{2}{3}}$$

(۸)

$$f(x)=x^2+ax+b \xrightarrow{(-1,-4)} \begin{cases} -4=1-a+b \\ -1=-1-a+b \end{cases} \Rightarrow a-b=3$$

$$-1-a+b=-4 \rightarrow b=2$$

$$-4+2+a=0 \Rightarrow a=-1$$

$$y=2x+a=0$$

$$y=2x+1$$

$$f(x)=x^3+x-2 \Rightarrow 2x+1=x^3+x-2$$

$$x^3-2x-1=0$$

$$x^3-sx+p \Rightarrow \begin{cases} s=1=-\frac{b}{a} \\ p=-1=\frac{c}{a} \end{cases}$$

$$(x+1)(x^2-x-1)$$

$x=-1$

$$\Delta=b^2-4ac$$

$$\Delta=1+4=5$$

$$x=\frac{+1\pm\sqrt{5}}{2}$$

$$\frac{1+\sqrt{5}}{2}+\frac{1-\sqrt{5}}{2}=\frac{2}{2}=1$$

x^3-2x-1 | $x+1$

x^3+x^2 | x^2-x-1

$-x^2-2x-1$

$-x^2-x$

$-x-1$

$-x-1$

0

(۹)

$$f=\{(2,a+b),(1,2a),(-1,a-2b+1)\}$$

$f(x)=k$

$$a+b=2a=a-2b+1$$

$$a+b=2a \Rightarrow b=a$$

$$a+b=a-2b+1$$

$$3b=1$$

$$b=\frac{1}{3}$$

$$\boxed{a=\frac{1}{3}}$$

(۱۰)

$$f(x) = \frac{4x^2 - ax + c + 1}{bx + 3}$$

$f(x) = x$ از ازای این عبارت باید یک x بشود

$$\frac{4x^2 - ax + c + 1}{bx + 3} = x$$

$$4x^2 - ax + c + 1 = bx^2 + 3x$$

$b = 4$

$$\frac{4x^2 - ax + c + 1}{x} = 4x - a$$

$c + 1$ باقی مانده $= 0$

$c = -1$

$$4x - a = bx + 3$$

$b = 4$
$a = -3$
$c = -1$