۱۹ آذر

۱. در ازای a $\rightarrow x^2+2x=ax-4$

$a^2+2a=a^2-4$

$2a=-4$

$a=-2$ ✓ ☺

۲. $f(x)=\frac{x^2+a}{2x-b}$ ، $g(x)=2x+b$

$\frac{x^2+a}{2x-b}=2x+b \xrightarrow{|\frac{1}{2}} \frac{4+a}{4-b}=4+b \Rightarrow 4+a=16-b^2$

$4+a=16$

$a=12$

$|\frac{1}{2}$ ↘ $g(x)=2x+b$

$2=4+b$

$-1=b$

$f(x)=\frac{x^2+12}{2x+1} \rightarrow f(1)=\frac{12}{3}=4$ ✓ ☺

۳. $f(x)=\frac{4x+1}{12x^2+ax+b}$ $D_f=R-\{-1,4\}$ $f(1)=?$

-1 و 4 ریشه‌های مخرج

$2x^2+ax+b\neq 0$

$2-a+b=0 \rightarrow 2+10+b=0$

$b=-12$

$32+4a+b=0$

$30+5a=0 \rightarrow a=-6$

$f(x)=\frac{4x+1}{12x^2-6x-18} \rightarrow f(1)=\frac{-5}{12}$ ✓ ☺

۴. $f(x)=\frac{x^2-\sqrt{2}}{-4x^2+ax+b}$ $R-\{1\}$ ریشه مضاعف

$-4x^2+ax+b=0$

$x=\frac{-b}{2a}=\frac{-a}{-8}=\frac{a}{8}=1 \rightarrow a=8$

$-4+a+b=0$

$b=-4$

$a+b=12$ ✓ ☺

۵. $f(x)=\frac{2x}{(x-1)(x^2+mx+1)}$ $R-\{1\}$

ریشه x ، ریشه $=1$

$\frac{-m}{2}=1 \Rightarrow m=-2$

$\Delta=m^2-4 \rightarrow m^2-4=0$

$m^2=4 \rightarrow m=\pm 2$

$m^2-4<0$

$m^2<4$

$-2<m<2$

$-2\leq m<2 \rightarrow m\in[-2,2)$ ✓ ☺

۶. $f(x)=\sqrt{4-\frac{1}{x^2}}$

$4-\frac{1}{x^2}\geq 0$

$(2+\frac{1}{x})(2-\frac{1}{x})\geq 0$

$\frac{-1}{2}$ $\frac{1}{2}$

$x\neq 0$

$D_f=(-\infty,-\frac{1}{2}]\cup[\frac{1}{2},+\infty)$ ✓ ☺

۷. $f(x)=\sqrt{mx^2+2mx+1}$

$m\in[0,1]$ ✓

$mx^2+2mx+1\geq 0 \rightarrow m>0$

$\Delta\leq 0 \Rightarrow 4m^2-4m\leq 0$

$4m(m-1)\leq 0$

$0\leq m\leq 1$ ✓ ☺

۸. $x=\frac{1}{2} \rightarrow g(x)=f(x)$

$2=2+k$

$0=k$

$a+k=\frac{1}{2}+0=+\frac{1}{2}$

$D_g=R \rightarrow D_f=R_1 \Rightarrow a=\frac{1}{2}$ ✓ ☺

$g_{(x)} = 3x + b$

$a = 1$
$b = 2$ $\Big\}$ $a - b = -1$

①

$x = \frac{-2}{3} \longrightarrow 3x + b = 3ax + 2$

$-2 + b = -2a + 2$

$a = 1$, b

$x = 2 \longrightarrow x + 2 = 2ax + a$

$4 = 2a + 2a$

$a = a^2 + a - 2$

$a = (a - 1)(a + 2)$

1 $\quad -2$

✓ ②

-i-

$$f(x)=\begin{cases}\dfrac{(3x+2)(3x-2)}{3x+2} & ; x\neq -\frac{2}{3}\\ 3ax+2 & ; x=-\frac{2}{3}\end{cases}$$

$\rightsquigarrow g(x)=3x+b$

$D_f=D_g=\mathbb{R} \rightsquigarrow 3x-2=3x+b \rightarrow \boxed{b=-2}$

$g(-\frac{2}{3})=3(-\frac{2}{3})-2=-4 \rightsquigarrow f(-\frac{2}{3})=-4$

$\hookrightarrow 3\times(-\frac{2}{3})(a)+2=-4 \rightarrow -2a=-6 \rightarrow \boxed{a=3}$

$\boxed{a-b=a}$