

شماره 28

۱۵ آزمون

(آیا حلقی - دهم دختر B)

$$\left. \begin{array}{l} x^2 + 12x \xrightarrow{x=a} a^2 + 12a \\ ax - 4 \xrightarrow{x=a} a^2 - 4 \end{array} \right\} \Rightarrow a^2 + 12a = a^2 - 4 \rightarrow a = \boxed{-1} \quad \text{سوال ۲}$$

$$f(x) = \frac{x^2 + a}{bx - b} \xrightarrow{(2,3)} \quad \text{و} \quad \frac{12}{12-b} = \frac{4+a}{4-b} \rightarrow 12 - 12b = 4 + a \quad \text{سوال ۲}$$

$$12 - 4 = a + 12b \Rightarrow 8 = a + 12b$$

$$g(x) = 2x + b \xrightarrow{(2,3)} \quad \text{و} \quad 3 = 4 + b \rightarrow b = \boxed{-1} \quad \text{و} \quad 8 = a - 3 \rightarrow a = \boxed{11} \quad \text{سوال ۲}$$

$$f(x) = \frac{x^2 + 11}{2x + 1} \Rightarrow f(1) = \frac{1 + 11}{2 + 1} = \frac{12}{3} = \boxed{4} \quad \text{و} \quad \text{سوال ۲}$$

$$\Rightarrow R - \{-1, 4\} \Rightarrow x_1 = -1, x_2 = 4 \quad \text{سوال ۳}$$

$$2x^2 + ax + b = 2(x - (-1))(x - 4) = 2(x + 1)(x - 4)$$

$$\Rightarrow 2x^2 - \frac{4x}{a} - \frac{b}{b} \Rightarrow a = -4, b = -8 \quad \text{سوال ۲}$$

$$f(x) = \frac{4x + 1}{2x^2 - 4x - 8} \rightarrow f(1) = \frac{4 + 1}{2 - 4 - 8} = \frac{5}{-10} = \boxed{-\frac{1}{2}} \quad \text{و} \quad \text{سوال ۲}$$

$$x = -1 \rightarrow -kx^2 + ax + b \Rightarrow -k(-1)^2 + a(-1) + b \quad \text{سوال ۴}$$

$$\downarrow$$

$$-k - a + b = 0 \rightarrow b - a = k$$

$$-k(x - (-1))^2 = 0 \rightarrow -k(x^2 + 2x + 1) = 0 \rightarrow -kx^2 - \frac{2kx}{a} - \frac{k}{b} = 0$$

$$a = -1, b = -2 \rightarrow a + b = -1 - 2 = \boxed{-3} \quad \text{و} \quad \text{سوال ۲}$$

$$\underbrace{(x^p + mx + 1)}_{(\Delta < 0) \text{ ليس له جذور حقيقية}} \cdot (x-1) \rightarrow \frac{(x-1)(x-1)(x-1)}{(x-1)^p} = (x-1)^{3-p} \quad (\text{سؤال ٥})$$

$$\textcircled{1} \Delta = b^p - 4ac = m^p - 4(1)(1) = m^p - 4 < 0 \rightarrow -2 < m < 2$$

$$\textcircled{2} x^p + mx + 1 \xrightarrow{x=1} 1 + m + 1 = 0 \rightarrow m + 2 = 0 \rightarrow m = -2$$

$$\Rightarrow -2 \leq m \leq 2 \quad \textcircled{2}$$

$$f(x) = \sqrt{k - \frac{1}{x^2}} \rightarrow \sqrt{k - \frac{1}{x^2}} \geq 0 \rightarrow \sqrt{(k - \frac{1}{x})(k + \frac{1}{x})} \geq 0 \quad (\text{سؤال ٤})$$

$$k - \frac{1}{x} \geq 0 \rightarrow k \leq \frac{1}{x} \quad k + \frac{1}{x} \geq 0 \rightarrow k \geq -\frac{1}{x}$$

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$$kx \leq 1 \rightarrow x \leq \frac{1}{k}$$

$$-kx \geq 1 \rightarrow x \geq -\frac{1}{k}$$

$$\frac{1}{k} \geq x \geq -\frac{1}{k} \quad \textcircled{1}$$

$$f(x) = \sqrt{mx^2 + px + 1} \quad (\text{سؤال ٧})$$

$$\textcircled{m=0} \rightarrow \frac{0}{x^2} + p \cdot \frac{1}{x} + 1 = 0 \rightarrow m = 1$$

$$\textcircled{m > 0} \rightarrow \Delta = b^p - 4ac \leq 0 \Rightarrow (pm)^p - 4(m)(1) = pm^p - 4m$$

$$\frac{pm(m-1)}{+} \leq 0 \quad \left. \begin{array}{l} m > 0 \rightarrow pm + \\ m-1 \geq 0 \rightarrow m \geq 1 \end{array} \right\} \Rightarrow m = [0, 1] \quad \textcircled{1}$$

$$f(x) \begin{cases} \frac{kx^2 - 1}{px - 1} : x \neq \frac{1}{p} \rightarrow \frac{(kx-1)(x+1)}{kx-1} & g(x) = px + 1 \\ kx + k : x = \frac{1}{p} \end{cases} \quad (\text{سؤال ٨})$$

$$f(x) = g(x) \Rightarrow x = \frac{1}{p} \quad f(1/p) = k(1/p) + k = p + k$$

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$$p + k = p \rightarrow k = 0$$

$$f(x) = px + 1$$

$$\rightarrow a = p, k = 0 \rightarrow a + k = p + 0 = p \quad \textcircled{1}$$

$$f(x) \begin{cases} \frac{x^p - f}{x + r} : x \neq -\frac{r}{p} \\ pax + r : x = -\frac{r}{p} \end{cases} \quad g(x) = px + b \quad \text{(سوال 9)}$$

$$\frac{(px - r)(pax + r)}{pax + r} \begin{cases} px - r \rightarrow px - r = px + b \Rightarrow b = -r \\ pax + r \end{cases}$$

$$f(x) = pa(-\frac{r}{p}) + r = -ra + r$$

$$g(x) = p(-\frac{r}{p}) + b = -r + (-r) = -2r \quad \begin{cases} -ra + r = -2r \\ -ra = -3r \Rightarrow a = 3 \end{cases}$$

$$a - b = 3 - (-2) = 5$$

$$f(x) \begin{cases} \frac{x^p - \varepsilon}{x - r} : x \neq r & \frac{(x-r)(x+r)}{x-r} \Rightarrow x+r \\ pa^p + ax : x = r \rightarrow pa^p + pa = \varepsilon \rightarrow pa^p + pa - \varepsilon = 0 \end{cases} \quad \text{(سوال 10)}$$

$$a^p + a - r = 0 \rightarrow (a+r)(a-1) = 0 \rightarrow a+r = 0 \Rightarrow a = -r \quad \checkmark$$

$$\hookrightarrow a-1 = 0 \Rightarrow a = 1 \quad \checkmark$$

$$f(x) = \sqrt{4 - \frac{1}{x^2}} \rightsquigarrow Df = 4 - \frac{1}{x^2} \geq 0$$

-4

$$\frac{1}{x^2} \leq \varepsilon \xrightarrow[\text{است جهت تغییرش کند}]{\text{چون مثبت}} \frac{x x^2}{x^2} \rightarrow 1 \leq 4x^2 \rightarrow x^2 \geq \frac{1}{4} \rightsquigarrow x \geq \frac{1}{2}$$

$$\rightsquigarrow x \leq -\frac{1}{2}$$

$$Df = (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, +\infty)$$

$$Dg = \mathbb{R} \rightsquigarrow Df = \mathbb{R} \rightsquigarrow a = \frac{1}{2} \quad \text{توجه! با بدنه } x = \frac{1}{2} \text{ تعریف نشود!} \quad -8$$

$$f(x) = \begin{cases} \frac{(x-1)(x+1)}{x-1} & ; x \neq \frac{1}{2} \\ x+k & ; x = \frac{1}{2} \end{cases}$$

$$g\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 2$$

$$\rightarrow f\left(\frac{1}{2}\right) = 2 \rightarrow 2\left(\frac{1}{2}\right) + k = 2 \rightarrow k = 0 \rightarrow a+k = \frac{1}{2} + 0 = \boxed{\frac{1}{2}}$$