۲۰ آذر

۱) ✓ ✓

$x^2 + 2a = x^2 - 8 \Rightarrow 2a = -8 \Rightarrow \boxed{a = -4}$

۱۲) ✓

$f(x) = \frac{x^2+a}{2x-b} = \frac{x^2+a}{2x+1} \xrightarrow{x=2} \frac{8+a}{5} = 3 \Rightarrow 8+a = 15 \Rightarrow a = 11$

$g(x) = 2x+b \xrightarrow{x=2} 8+b = 3 \Rightarrow b = -1$

$f(1) = \frac{1+11}{2+1} = \frac{12}{3} = \boxed{4}$

۳) ✓

$f(x) = \frac{8x+1}{2x^2+ax+b}$

$D_f = \mathbb{R} - \{-1, 4\}$ (ریشه‌های مخرج)

$\Rightarrow -\frac{a}{2} = 3 \Rightarrow a = -6$ ، $\frac{b}{2} = -4 \Rightarrow b = -8$

$\Rightarrow f(x) = \frac{8x+1}{2x^2-6x-8} \xrightarrow{x=1} f(1) = \frac{8+1}{-12} = \boxed{\frac{-9}{12}}$

۴) ✓

$f(x) = \frac{x^3 - \sqrt{3}}{-3x^2+ax+b}$

$D_f = \mathbb{R} - \{-1\}$

ریشه‌ی مخرج $\Rightarrow \Delta = 0 \Rightarrow a^2 + 12b = 0$

$\frac{-a}{-3} = \frac{a}{3} = -2$ ، $\frac{-b}{3} = 1$

$\Rightarrow a = -6$ ، $b = -3$

$\Rightarrow a+b = -6-3 = \boxed{-9}$

۵) ✓

$f(x) = \frac{x}{(x-1)(x^2+mx+1)}$

$D_f = \mathbb{R} - \{1\}$

① x^2+mx+1 ریشه ندارد $\Rightarrow \Delta < 0 \Rightarrow m^2 - 4 < 0 \Rightarrow m^2 < 4 \Rightarrow -2 < m < 2$

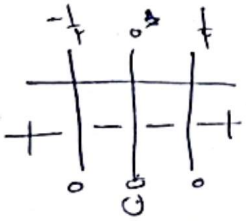
② x^2+mx+1 یک ریشه‌ی مضاعف دارد $\Rightarrow -\frac{m}{1} = 2 \Rightarrow m = -2$

$\boxed{-2 \leq m < 2}$

۶) ✓

$f(x) = \sqrt{3 - \frac{1}{x^2}}$

$3 - \frac{1}{x^2} \geq 0 \Rightarrow \frac{3x^2-1}{x^2} \geq 0$ ، $\frac{(2x-1)(2x+1)}{x^2} \geq 0$



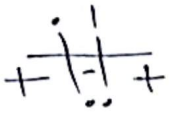
$\Rightarrow D_f = (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, +\infty) = \mathbb{R} - (-\frac{1}{2}, \frac{1}{2})$

۷) ✓

$f(x) = \sqrt{mx^2+2mx+1}$

$mx^2 + 2mx + 1 \geq 0$ (همواره نامنفی) $\Rightarrow m \geq 0$ I (چون نقطه‌ی خاصی ندارد)

$\Delta \leq 0 \Rightarrow 4m^2 - 4m \leq 0$ ، $4m(m-1) \leq 0$



$0 < m \leq 1$ II $\subset$

$\Rightarrow I \cap II = 0 < m \leq 1$

$0 \leq m \leq 1$ ← تابع ثابت $f(x) = 1$ برای $m = 0$

۸) ✓

$f(x) = \begin{cases} \frac{4x^2-1}{2x-1} & ; x \neq a \Rightarrow a = \frac{1}{4} \\ 4x+k & ; x = \frac{1}{4} \end{cases}$

$\frac{4x^2-1}{2x-1} = 2x+1$ ✓

$x = \frac{1}{4} \Rightarrow 2 + k = 2 \Rightarrow k = 0$

$\Rightarrow a + k = \frac{1}{4} + 0 = \boxed{\frac{1}{4}}$

$$\frac{9x^2-4}{3x+2} = \frac{(3x-2)(3x+2)}{3x+2} = 3x-2 = 3x+b \Rightarrow b=-2$$

$$x=-\frac{2}{3} \Rightarrow -2a+2 = -2-2 \Rightarrow 1-a=-2 \Rightarrow a=3$$

$$\Rightarrow a-b = 3+2 = 5$$

$$x=2 \Rightarrow 2a^2+2a = 4 \qquad a^2+a-2=0 \qquad (a+2)(a-1)=0 \begin{cases} a=-2 \\ a=1 \end{cases}$$

$$a=-2, 1$$