

$$\text{ii) } y = \sqrt{x} - \sqrt{y-x} = y-x \geq 0 \Rightarrow x \leq y, \quad x - \sqrt{y-x} \geq 0 \Rightarrow \sqrt{y-x} \leq x \Rightarrow y-x \leq x^2$$

$$x \geq -1 \Rightarrow D_f = [-1, y]$$

$$\text{b) } \sqrt{r - \sqrt{x-r}} \geq 0 \Rightarrow x-r \geq 0 \Rightarrow x \geq r, \quad r - \sqrt{x-r} \geq 0 \Rightarrow \sqrt{x-r} \leq r \Rightarrow x-r \leq r^2$$

$$x \leq 11 \Rightarrow D_f = [r, 11]$$

الف) $\sqrt{1-x^2} \Rightarrow y(1-x^2) \Rightarrow \frac{-\sqrt{1-x^2} + \sqrt{1-x^2}}{-1+1} \Rightarrow D_f = [-1, 1]$

ب) $\sqrt{r|x|-9} \Rightarrow r(|x|-r) \Rightarrow$

$\begin{array}{c|c|c} -r & +r \\ \hline + & - & + \end{array}$

$\Rightarrow D_f = (-\infty, -r] \cup [r, +\infty)$

لـ $r|x|-9 \geq 0 \Rightarrow r|x| \geq 9 \Rightarrow |x| \geq r \Rightarrow x \geq r$
 $x \leq -r$

$$\lim_{x \rightarrow \infty} \sqrt{\frac{|x|+1}{|x|-3}} \Rightarrow |x|-3 \neq 0 \Rightarrow |x| \neq 3 \Rightarrow x \neq \pm 3 \Rightarrow D_f = \mathbb{R} - \{\pm 3\}$$

$$\text{ب. } \sqrt{\frac{\sqrt{x}+1}{\sqrt{x}-1}} \Rightarrow \begin{cases} \sqrt{x} \geq 0 \Rightarrow x \geq 0 \text{ I} \\ \sqrt{x}-1 \neq 0 \Rightarrow \sqrt{x} \neq 1 \Rightarrow x \neq 1 \text{ II} \end{cases} \Rightarrow D_f = [0, +\infty) - \{1\}$$

$$\text{الف) } \frac{\sqrt{3-|x|}}{|x|+2} = 3-|x| \geq 0 \Rightarrow |x| \leq 3 \Rightarrow -3 \leq x \leq 3 \Rightarrow D_f = [-3, 3]$$

ب) $\frac{\sqrt{r-x^2}}{|x|-1} = r \cdot x^r \Rightarrow \frac{-r}{-1} \frac{1}{x^2} = -r \Rightarrow [-r, r]$
 $|x|-1 \neq 0 \Rightarrow |x| \neq 1 \Rightarrow x \neq \pm 1$ II

الف) $\frac{x+1}{\sqrt{x+|x|}} \Rightarrow x+|x| > 0 \Rightarrow \begin{matrix} + \rightarrow \checkmark \\ 0 \rightarrow \times \\ - \rightarrow \times \end{matrix} \Rightarrow D_f = \mathbb{R}^+$

ج) $\frac{1}{\sqrt{x|x|}} \Rightarrow x|x| > 0 \Rightarrow \begin{matrix} \nearrow + \rightarrow \checkmark \\ \rightarrow 0 \rightarrow \times \\ \searrow - \rightarrow \times \end{matrix} \Rightarrow D_f = \mathbb{R}^+$

$$\text{الف) } \sqrt{r-x} \Rightarrow r-x \geq 0 \Rightarrow x \leq r \Rightarrow D_f = (-\infty, r)$$

$$\text{ب) } \frac{1}{\sqrt{r-x}} \Rightarrow r-x > 0 \Rightarrow x < r \Rightarrow D_f = (-\infty, r)$$

$$\text{الف) } \frac{1}{x[x]} \Rightarrow x[x] \neq 0 \Rightarrow x, [x] \neq 0 \Rightarrow D_f = \mathbb{R} - [0, 1)$$

$$\text{ب) } \frac{1}{\sqrt{-x[x]}} \Rightarrow -x[x] > 0 \Rightarrow -x, [x] > 0 \Rightarrow \begin{matrix} + \rightarrow x \\ 0 \rightarrow x \\ - \rightarrow x \end{matrix} \Rightarrow D_f = \emptyset$$

$$\text{الف) } \sqrt{\left[x - \frac{1}{r}\right] + \left[x + \frac{r}{r}\right]} \Rightarrow \left[x + \frac{r}{r} - 1\right] + \left[x + \frac{r}{r}\right] \geq 1 \Rightarrow r\left[x + \frac{r}{r}\right] \geq 1 \Rightarrow \left[x + \frac{r}{r}\right] \geq \frac{1}{r}$$

$$x + \frac{r}{r} \geq 1 \Rightarrow x \geq \frac{1}{r} \Rightarrow D_f = \left\{\frac{1}{r}, +\infty\right\}$$

$$\text{ب) } \sqrt{\left[x - \frac{1}{r}\right] + \left[-x + \frac{1}{r}\right]} \Rightarrow \begin{cases} a \in \mathbb{Z} & -[a] + [-a] = 0 \\ a \in \mathbb{Z} & -[a] + [-a] = -1 \end{cases} \Rightarrow x - \frac{1}{r} \in \mathbb{Z} \Rightarrow D_f = \left\{x \mid x = k + \frac{1}{r}, k \in \mathbb{Z}\right\}$$

$$\text{الف) } \frac{1}{r \sin^2 x - 1} \Rightarrow r \sin^2 x - 1 \neq 0 \Rightarrow r \sin^2 x \neq 1 \Rightarrow \sin^2 x \neq \frac{1}{r} \Rightarrow \sin x \neq \pm \frac{1}{\sqrt{r}}$$

$$D_f = \mathbb{R} - \left\{\pm \frac{1}{\sqrt{r}}\right\}$$

$$\text{ب) } \frac{\cot x + 1}{\tan x + 1} \Rightarrow \frac{\frac{\cos}{\sin} + 1}{\frac{\sin}{\cos} + 1} \Rightarrow \begin{matrix} \tan \neq -1 \Rightarrow 135^\circ, 225^\circ \\ \cos \neq 0 \Rightarrow 90^\circ, 270^\circ \end{matrix} \Rightarrow D_f = \mathbb{R} - \left\{k\pi + \frac{\pi}{4}, k\pi - \frac{\pi}{4}\right\}$$



$$\text{الف) } \sqrt{r \sin x - 1} \Rightarrow r \sin x - 1 \geq 0 \Rightarrow r \sin x \geq 1 \Rightarrow \sin x \geq \frac{1}{r} \Rightarrow D_f = \left[k\pi + \frac{\pi}{4}, k\pi + \frac{3\pi}{4}\right]$$



$$\text{ب) } \sqrt{1 - r \cos x} = 1 - r \cos x \geq 0 \Rightarrow r \cos x \leq 1 \Rightarrow \cos x \leq \frac{1}{r} \Rightarrow D_f = \mathbb{R} - \left(k\pi + \frac{\pi}{4}, k\pi - \frac{\pi}{4}\right)$$

