۱

(۲) الف) $x|x| \geq 0 \rightarrow \begin{matrix} + \checkmark \\ - \times \\ 0 \checkmark \end{matrix} \Rightarrow D_f = [0, +\infty)$ ⇒ وارون‌پذیر

$D_g = \mathbb{R}$

ب) $D_f = \mathbb{R}$ ← $\Delta < 0$ ← مخرج کسر در $f(x)$ تا قدرمطلق حقیقی است ⇒ وارون‌پذیر

$D_g = \mathbb{R}$

ج) $D_g = \mathbb{R}$

$2\sin x \neq 3 \rightarrow \sin x \neq \frac{3}{2}$ ← $\frac{3}{2}$ بزرگتر از ۱ است و نمی‌شود

$\Rightarrow D_f = \mathbb{R}$ ← دامنه D_f ، D_g ← وارون‌پذیر

د) $|x| \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\}$

$x \neq 0 \Rightarrow D_g = \mathbb{R} - \{0\}$ ← وارون‌پذیر

۲

(۲) الف) $f(x) = \left\{\frac{x^2+1-1}{x^2+1}\right\} = \left[1 - \frac{1}{x^2+1}\right] = [1^-] = 0 \Rightarrow \boxed{D_f = \mathbb{R}}$

$g(x) = [x - [x]] = 0 \Rightarrow \boxed{D_g = \mathbb{R}}$ ⇒ این ۲ تابع وارون‌پذیر

ب) $f(x) \rightarrow [2x] \neq 0 \rightarrow D_f = \mathbb{R} - [0, 0.5)$

$2[x] \neq 0 \rightarrow [x] \neq 0 \Rightarrow D_g = \mathbb{R} - [0, 1)$ ← وارون‌پذیر

ج) $x - |x| > 0 \rightarrow x > |x| \overset{\begin{matrix}+x \\ -x \\ 0x\end{matrix}}{\Rightarrow} D_f = \emptyset$

$|x| - x > 0 \rightarrow |x| > x \begin{matrix} + x \\ - \checkmark \\ 0 x \end{matrix} \Rightarrow D_g = \mathbb{R}^-$ یا $(-\infty, 0)$ یا $0 > x$ ← وارون‌پذیر

د) $D_g = \mathbb{R}$

$D_f = \mathbb{R}$ ← وارون‌پذیر

۳

$(g(x) + f(x)) + (f(x) - g(x)) = 2f(x) = x^2 + x + 1 + x^2 - x + 1 = 2x^2 + 2 = 2f(x)$ (۲)

$g(x) + f(x) = x^2 + x + 1 \Rightarrow g(x) = x$

$x^2 + 1$

$\Leftarrow \boxed{f(x) = x^2 + 1}$

$\Rightarrow f^2(x) - g^2(x) = (x^2+1)^2 - (x)^2 = x^4 + 2x^2 + 1 - x^2 = x^4 + x^2 + 1$

$\mathbb{R}$ ← دامنه

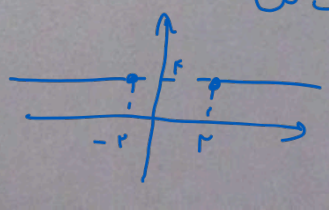
۴

$f \times g = (x - \sqrt{x^2-4})(x + \sqrt{x^2-4}) = x^2 + x\sqrt{x^2-4} - x\sqrt{x^2-4} - x^2 - 4 =$ (۴)

(۱) دقت کن (-۴)

$x^2 - (x^2 - 4) = x^2 - x^2 + 4 < 4$

$D_{f \times g} = (-\infty, -2] \cup [2, +\infty)$



۵)

$$\frac{(x-b)\div 2}{(2x^2-3x-5)\div 2} = \frac{\frac{x}{2}-\frac{b}{2}}{x^2-\frac{3}{2}x-\frac{5}{2}} = \frac{ax+2}{x^2-mx+n}$$

$$\frac{x}{2}-\frac{b}{2} = ax+2 \Rightarrow \boxed{a=\frac{1}{2}}\ ,\ \boxed{b=-4}$$

$$x^2-\frac{3}{2}x-\frac{5}{2} = x^2-mx+n$$

$$\boxed{n=\frac{-5}{2}}$$

$$\frac{-3}{2}x = -mx$$

$$\boxed{\frac{3}{2}=m}$$

$$\rightarrow am-bn = \left(\frac{1}{2}\times\frac{3}{2}\right)-\left(-4\times\frac{-5}{2}\right) = \frac{3}{4}-10 = \boxed{\frac{-37}{4}}$$

۶)

$$8x+b\neq 0 \Rightarrow x\neq\frac{-b}{8} \Rightarrow \frac{-b}{8}\neq a \rightarrow a\neq -\frac{b}{8}$$

$$\frac{bx+2}{8x+b} = c \rightarrow 8cx+bc = bx+2$$

$$bc=2 \leftarrow \frac{b^2}{8}=2$$

$$8cx = bx$$

$$b=\pm 4$$

$$8c = b$$

$$b=8c \leftarrow b=-4 \leftarrow \frac{-1}{2}$$

$$c=\frac{b}{8}$$

$$\hookrightarrow b=4 \rightarrow c=\frac{1}{2}$$

$$\frac{ab}{c} = \begin{cases} \frac{\frac{-1}{2}\times 4}{\frac{1}{2}} = -4 \\ \frac{\frac{1}{2}\times -4}{\frac{-1}{2}} = +4 \end{cases}$$

۷)

$x=2,3,-1$ → نمره

$$2f-1 \xrightarrow{x=2} 2f-1=7 \Rightarrow \boxed{f=4}\ ,\ 2f-1 \xrightarrow{x=3} 2f-1=-5 \Rightarrow \boxed{f=-2}\ ,\ 2f-1 \xrightarrow{x=-1} 2f-1=3 \Rightarrow \boxed{f=2}$$

$$g(x) = \{(2,4)(3,9)(-1,-4)\} \qquad f(x) = \{(2,4)(3,-2)(-1,2)\}$$

$$\rightarrow \frac{2g}{f+g} = \left\{\left(2,\frac{8}{8}\right)\left(3,\frac{12}{6}\right)\left(-1,\frac{4}{-2}\right)\right\} = \{(2,1)(3,3)(-1,-2)\}$$

۸)

$$\boxed{a=d=-1}$$

$$3a-2b=1 \qquad a-3b=c$$

$$-3-2b=1 \qquad -1+9=c \Rightarrow d+c=-1+9=8$$

$$-4=2b \qquad \boxed{9=c}$$

$$\boxed{-2=b}$$

۹)

$$f(x) = \sqrt{-(x^2-x+m)} \Rightarrow$$

$g(x)$ فقط در یک نقطه پاسخ دارد.

$$\frac{\Delta}{b^2-4ac} = 1-4m=0 \rightarrow m=\frac{1}{4} \Rightarrow f(x)=\sqrt{-\left(x^2-x+\frac{1}{4}\right)}$$

$$f(x) = \sqrt{-(x-\frac{1}{2})^2} \Rightarrow x=\frac{1}{2}$$

$$a+b = \frac{1}{2}+0 = \frac{1}{2}$$

$$f(x)=0 \leftarrow \{(a,b)\} = \leftarrow$$

۱۰)

$$\frac{2}{x-c} = \frac{4x+a}{x^2+4x+b} \rightarrow (x-c)(x+d) = x^2+4x+b \rightarrow \frac{2}{x-c} = \frac{4x+a}{(x-c)(x+d)}$$

$$4x+a = 2(x+d)$$

$$a = 2(4+c) = 12+2c$$

$$2d = a$$

$$x^2+x(d-c)-cd = x^2+4x+b$$

$$d-c=4\ ,\ -cd=b \rightarrow 4d-d^2=b$$

$$d=4+c\ ,\ -c=4-d$$

$$\begin{cases} a+b+c = 12+2c-c^2-4c+c \\ c^2-3c+12=? \\ a+b+c=12 \Leftarrow c=-3 \text{ اثبات} \end{cases}$$