

1.
$$\text{الف) } y = \sqrt{\frac{x^2 - 4x}{x^2 - x}} \Rightarrow D_f = [4, +\infty)$$

$$x^2 - x \neq 0$$

$$x^2(x-1) \neq 0 \Rightarrow x \neq 0, 1, -1$$

$$\frac{x^2 - 4x}{x^2 - x} > 0$$

$$\Rightarrow x^2 - 4x > 0 \Rightarrow x^2 > 4x \Rightarrow x > 4$$

2.
$$\text{ب) } y = \sqrt{\frac{x^5 - x^3}{x^2 - x}} \quad D_f = \mathbb{R} - \{-1, 0, 1\}$$

$$x^2 - x \neq 0 \Rightarrow x \neq 0, 1, -1$$

$$\frac{x^5 - x^3}{x^2 - x} > 0$$

$$\Rightarrow x^5 - x^3 > 0 \Rightarrow x^2 > 1$$

$$x > 1$$

$$x < -1$$

3.
$$\text{الف) } y = \frac{x+1}{\sqrt{x+|x|}}$$

$$\Rightarrow x + |x| > 0 \Rightarrow |x| > -x \Rightarrow$$

$$D_f = \mathbb{R}^+$$

$$(0, +\infty)$$

ب)
$$y = \frac{1}{\sqrt{x|x|}}$$

$$x|x| > 0 \Rightarrow D_f = \mathbb{R}^+$$

$$(0, +\infty)$$

4.
$$\text{الف) } y = \sqrt{2 - [x]}$$

$$\Rightarrow 2 - [x] > 0 \Rightarrow [x] < 2 \Rightarrow$$

$$x < 3$$

$$D_f = (-\infty, 3)$$

ب)
$$y = \frac{1}{\sqrt{2 - [x]}}$$

$$\Rightarrow 2 - [x] > 0 \Rightarrow [x] < 2 \Rightarrow x < 3$$

$$(-\infty, 3)$$

أ) $y = \sqrt{\left[n - \frac{1}{r}\right] + \left[n + \frac{r}{r}\right]}$

$\left[n - \frac{1}{r}\right] + \left[n + \frac{r}{r}\right] \geq 0 \quad \left[n + \frac{r}{r} - 1\right] + \left[n + \frac{r}{r}\right] \geq 0$

$\left[n + \frac{r}{r}\right] - 1 + \left[n + \frac{r}{r}\right] \geq 0 \Rightarrow 2\left[n + \frac{r}{r}\right] \geq 1 \Rightarrow \left[n + \frac{r}{r}\right] \geq \frac{1}{2}$

$n + \frac{r}{r} \geq 1 \Rightarrow n \geq \frac{1}{r} \Rightarrow D_f = \left[\frac{1}{r}, +\infty\right)$

ب) $y = \sqrt{\left[n - \frac{1}{r}\right] + \left[-n + \frac{1}{r}\right]}$

$\sqrt{\left[n - \frac{1}{r}\right] + \left[-\left(n - \frac{1}{r}\right)\right]} \xrightarrow{it} \begin{cases} a \in \mathbb{Z} & [a] + [-a] = 0 \\ a \notin \mathbb{Z} & [a] + [-a] = -1 \end{cases}$

$n - \frac{1}{r} \in \mathbb{Z} \Rightarrow D_f = \left\{n \mid n = p + \frac{1}{r}, p \in \mathbb{Z}\right\}$

أ) $y = \frac{r}{\sin n - 1}$

$\sin n - 1 \neq 0 \Rightarrow \sin n \neq 1 \Rightarrow$

$D_f = \mathbb{R} - \left\{2k\pi + \frac{\pi}{2}\right\}$

ب) $y = \frac{r}{r \sin^2 n - 1}$

$r \sin^2 n - 1 \neq 0 \Rightarrow r \sin^2 n \neq 1 \Rightarrow$

$\sin n \neq \frac{1}{r}, \sin n \neq -\frac{1}{r}$

$D_f = \mathbb{R} - \left\{k\pi \pm \frac{\pi}{r}\right\}$

أ) $y = \sqrt{r \sin n - 1}$

$r \sin n - 1 \geq 0 \Rightarrow$

$r \sin n \geq 1 \Rightarrow \sin n \geq \frac{1}{r}$

$D_f = \left[2k\pi + \frac{\pi}{r}, 2k\pi + \frac{\pi}{r}\right]$



$$b) \sqrt{\cos x - 1} \Rightarrow \cos x - 1 \geq 0 \Rightarrow \cos x \geq 1$$

$$D_f = \mathbb{R} - \{2k\pi\}$$

$$c) \cos y = |x| - 3 \quad -1 \leq |x| - 3 \leq 1$$

$$D_f = [-3, 2] \cup [2, 4] \quad 2 \leq |x| \leq 4 \quad \begin{matrix} -4 \leq x \leq -2 \\ 2 \leq x \leq 4 \end{matrix}$$

$$[-4, 4]$$

$$d) y = \arcsin(x^2 + 3x + 1)$$

$$-1 \leq x^2 + 3x + 1 \leq 1 \Rightarrow -1 \leq x^2 + 3x + 1$$

$$\downarrow$$

$$x^2 + 3x + 1 \leq 1$$

$$x^2 + 3x \leq 0$$

$$x(x+3) \leq 0$$

$$x \leq -3$$

$$-2 \leq x^2 + 3x$$

$$-2 \leq x(x+3)$$

$$[-1, +\infty)$$

$$e) \sqrt{2 - \log_r(x-2)} \quad \log_r \frac{(x-2)}{r} \leq 2$$

$$D_f = (2, 11] \quad x-2 \leq 9 \Rightarrow 2 \leq x \leq 11$$

$$f) y = \log(x^2 - x) \quad x^2 - x > 0 \Rightarrow x^2 > x \Rightarrow$$

$$D_f = \mathbb{R}^+$$

$$\text{ب) } \sqrt{\cos x - 1} \Rightarrow \cos x - 1 \geq 0 \Rightarrow \cos x \geq 1$$

$$D_f = \mathbb{R} - \{2k\pi\}$$

$$\text{ان) } \cos y = |x| - 2 \quad -1 \leq |x| - 2 \leq 1$$

$$D_f = [-2, 2] \cup [2, 4] \quad 2 \leq |x| \leq 4 \quad -2 \leq x \leq 2$$

$$[-2, 2] \quad 2 \leq x \leq 4$$

$$\text{ب) } y = \arcsin(x^2 + 3x + 1)$$

$$-1 \leq x^2 + 3x + 1 \leq 1 \Rightarrow -1 \leq x^2 + 3x + 1$$

$$x^2 + 3x + 1 \leq 1$$

$$x^2 + 3x \leq 0$$

$$x(x+3) \leq 0$$

$$x \leq -3$$

$$-2 \leq x^2 + 3x$$

$$-2 \leq x(x+3)$$

$$[-1, +\infty)$$

$$\text{ان) } \sqrt{2 - (\log_r(x-2))} \quad \log_r(x-2) \leq 2$$

$$D_f = (2, 11] \quad x-2 \leq 9 \Rightarrow 2 \leq x \leq 11$$

$$\text{ب) } y = \log(x^2 - x)$$

$$x^2 - x > 0 \Rightarrow x^2 > x \Rightarrow D_f = \mathbb{R}^+$$

$$a) y = \frac{r}{r^n + 1}$$

$$r^n + 1 \neq 0 \Rightarrow r^n \neq -1$$

$$\log_r^{-1} = n \rightarrow \text{مستبعد} \Rightarrow D_f = \mathbb{R}$$

$$b) y = \frac{r}{r^n - 1}$$

$$r^n - 1 \neq 0 \Rightarrow r^n \neq 1 \Rightarrow \log_r^1$$

$$D_f = \mathbb{R} - \{\log_r^1\}$$

$$c) y = \frac{r}{r^n - r}$$

$$r^n - r \neq 0 \Rightarrow r^n \neq r \Rightarrow \log_r^r$$

$$D_f = \mathbb{R} - \{\log_r^r\}$$

$$d) y = \frac{r}{r^n - r}$$

$$r^n - r \neq 0 \Rightarrow r^n \neq r \Rightarrow D_f = \mathbb{R} - \{\log_r^r\}$$

$$y = (r^n + 1)!$$

$$r^n + 1 \in \mathbb{W} \Rightarrow r^n \in \mathbb{W} - 1 \Rightarrow n \in \frac{\mathbb{W} - 1}{r}$$

$$y = \left(\frac{r^n - r}{r^n - a} \right)!$$

$$\frac{r^n - r}{r^n - a} \in \mathbb{W} \Rightarrow r^n - r \in (r^n - a)\mathbb{W}$$

$$r^n \in (r^n - a)\mathbb{W} + r \Rightarrow$$

$$n \in \frac{(r^n - a)\mathbb{W} + r}{r}$$