

الف) $y = \sqrt{\frac{x^2 - 2x}{x^3 - x}} = \sqrt{\frac{x(x-2)}{x(x+1)(x-1)}}$ $D_f = (-1, 0) \cup (0, 1) \cup [2, +\infty)$

ب) $y = \sqrt{\frac{x^5 - 2x^3}{x^3 - x}} = \sqrt{\frac{x^3(x+1)(x-1)}{x(x+1)(x-1)}}$ $D_f = (-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, +\infty)$

الف) $y = \frac{x+1}{\sqrt{x+|x|}}$ $x + |x| > 0 \Rightarrow D_f = (0, +\infty) = \mathbb{R}^+$

ب) $y = \frac{1}{\sqrt{x|x|}}$ $x|x| > 0 \Rightarrow D_f = (0, +\infty) = \mathbb{R}^+$

الف) $y = \sqrt{2 - [x]}$ $2 - [x] \geq 0 \Rightarrow 2 \geq [x] \Rightarrow D_f = (-\infty, 3)$

ب) $y = \frac{1}{\sqrt{2 - [x]}}$ $2 - [x] > 0 \Rightarrow 2 > [x] \Rightarrow D_f = (-\infty, 2)$

الف) $y = \sqrt{[x - \frac{1}{3}] + [x + \frac{2}{3}]}$ $[x - \frac{1}{3}] + [x + \frac{2}{3}] \geq 0 \Rightarrow 1$

ب) $y = \sqrt{[x - \frac{1}{3}] + [-x + \frac{1}{3}]}$ $[x - \frac{1}{3}] + [-x + \frac{1}{3}] \geq 0 \Rightarrow 0$

$y = [a] + [-a] \xrightarrow{\text{if}} \begin{cases} a \in \mathbb{Z} & [a] + [-a] = 0 \\ a \notin \mathbb{Z} & [a] + [-a] = -1 \end{cases} \Rightarrow x - \frac{1}{3} \in \mathbb{Z} \Rightarrow x \in \mathbb{Z} + \frac{1}{3}$

$D_f = \{x \mid x = k + \frac{1}{3}, k \in \mathbb{Z}\}$

الف) $y = \frac{2}{\sin x - 1}$ $\sin x - 1 \neq 0 \Rightarrow \sin x \neq 1 \Rightarrow D_f = \mathbb{R} - \{2k\pi + \frac{\pi}{2}\}$

ب) $y = \frac{3}{2\sin^2 x - 1}$ $2\sin^2 x - 1 \neq 0 \Rightarrow \sin^2 x \neq \frac{1}{2} \Rightarrow \sin x \neq \pm \frac{1}{\sqrt{2}} \Rightarrow D_f = \mathbb{R} - \{2k\pi + \frac{\pi}{4}, 2k\pi + \frac{3\pi}{4}\}$

الف) $y = \sqrt{2\sin x - 1}$ $2\sin x - 1 \geq 0 \rightarrow 2\sin x \geq 1 \rightarrow \sin x \geq \frac{1}{2}$
 $D_f = [2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}]$

ب) $y = \sqrt{\cos x - 1}$ $\cos x - 1 \geq 0 \rightarrow \cos x \geq 1$ $D_f = 2k\pi$

الف) $\cos y = |x| - 3$ $-1 \leq |x| - 3 \leq 1$
 $2 \leq |x| \leq 4 \rightarrow \begin{cases} 2 \leq x \leq 4 \\ -4 \leq x \leq -2 \end{cases}$ $D_f = [-4, -2] \cup [2, 4]$

ب) $y = \arcsin(x^2 + 3x + 1)$ $D_f = (I) \cap (II) \Rightarrow D_f = [-2, -1] \cup [-1, 0]$
 $-1 \leq x^2 + 3x + 1 \leq 1 \rightarrow \begin{cases} x^2 + 3x + 1 \leq 1 \rightarrow x^2 + 3x \leq 0 \rightarrow x(x+3) \leq 0 \\ x^2 + 3x + 1 \geq -1 \rightarrow x^2 + 3x + 2 \geq 0 \rightarrow (x+1)(x+2) \geq 0 \end{cases}$
 $\begin{matrix} -3 & 0 & 0 & + \\ + & 0 & - & 0 & + \end{matrix} (I)$
 $\begin{matrix} -2 & -1 & 0 & + \\ + & 0 & - & 0 & + \end{matrix} (II)$

الف) $y = \sqrt{2 - \log_3(x-2)}$ $x-2 > 0$ $2 - \log_3(x-2) \geq 0 \rightarrow 2 \geq \log_3(x-2)$
 $\boxed{x > 2}$ $\boxed{9 \geq x-2}$
 $D_f = (2, 11]$

ب) $y = \log(x^3 - x)$ $x^3 - x > 0$
 $x(x+1)(x-1) > 0$ $\begin{matrix} -1 & 0 & 1 \\ + & - & + \end{matrix}$ $D_f = (-1, 0) \cup (1, +\infty)$

الف) $y = \frac{3}{4^x + 1}$ $4^x + 1 \neq 0 \rightarrow 4^x \neq -1$ $D_f = \mathbb{R}$

ب) $y = \frac{3}{4^x - 1}$ $4^x - 1 \neq 0 \rightarrow 4^x \neq 1 \rightarrow x \neq 0$ $D_f = \mathbb{R} - \{0\}$

ج) $y = \frac{3}{4^x - 2}$ $4^x - 2 \neq 0 \rightarrow 4^x \neq 2 \rightarrow x \neq \frac{1}{2}$ $D_f = \mathbb{R} - \{\frac{1}{2}\}$

د) $y = \frac{3}{4^x - 3}$ $4^x - 3 \neq 0 \rightarrow 4^x \neq 3 \rightarrow x \neq \log_4 3$ $D_f = \mathbb{R} - \{\log_4 3\}$

الف) $y = (4x+1)!$ $4x+1 \in \mathbb{W} \Rightarrow 4x \in \mathbb{W} - 1 \Rightarrow x \in \frac{\mathbb{W}-1}{4}$
 $D_f = \{x \mid x = \frac{k-1}{4}, k \in \mathbb{W}\}$

ب) $y = \left(\frac{3x-2}{2x-5}\right)!$ $\frac{3x-2}{2x-5} \in \mathbb{W} \rightarrow x = -\frac{5\mathbb{W}+2}{2\mathbb{W}-3} = \frac{5\mathbb{W}-2}{2\mathbb{W}-3}$
 $D_f = \{x \mid x = \frac{5k-2}{2k-3}, k \in \mathbb{W}\}$