

الف) $y = \sqrt{\frac{x^2 - 4x}{x^2 - x}}$
 $\frac{x^2 - 4x}{x^2 - x} \geq 0 \rightarrow \frac{x(x-4)}{x(x-1)(x+1)} \geq 0$
 Sign chart: $-1 \quad 0 \quad 1 \quad 4$
 $D_f = (-1, 0) \cup (0, 1) \cup [4, +\infty)$
 (✓) ✓

ب) $y = \sqrt{\frac{x^2 - x^2}{x^2 - x}}$
 $\frac{x^2 - x^2}{x^2 - x} \geq 0 \rightarrow \frac{x^2(x-1)(x+1)}{x(x-1)(x+1)} \geq 0$
 Sign chart: $-1 \quad 0 \quad 1$
 $D_f = \mathbb{R} - \{+1, 0\}$

الف) $y = \frac{x+1}{\sqrt{x+|x|}}$
 $x+|x| > 0 \rightarrow x > 0$
 ① $x+x > 0 \rightarrow 2x > 0 \Rightarrow x > 0$
 ② $x-x > 0 \rightarrow 0 > 0$
 $D_f = (0, +\infty)$
 (✓) ✓

ب) $y = \frac{1}{\sqrt{x|x|}}$
 $x|x| > 0 \rightarrow x > 0$
 ① $x(x) > 0 \rightarrow x^2 > 0 \Rightarrow x > 0$
 ② $x(-x) > 0 \rightarrow -x^2 > 0 \times$
 $D_f = (0, +\infty)$

الف) $y = \sqrt{2 - [x]}$
 $2 - [x] \geq 0 \rightarrow 2 \geq [x]$
 $D_f = (-\infty, 2)$
 (✓) ✓

ب) $y = \frac{1}{\sqrt{2 - [x]}}$
 $2 - [x] > 0 \rightarrow 2 > [x]$
 $D_f = (-\infty, 2)$

الف) $y = \sqrt{[x - \frac{1}{x}] + [x + \frac{x}{x}]} \rightarrow [x - \frac{1}{x} + 1]$
 $\Rightarrow [x - \frac{1}{x}] + [x - \frac{1}{x}] + 1 \geq 0 \rightarrow 2[x - \frac{1}{x}] \geq -1$
 $\Rightarrow [x - \frac{1}{x}] \geq -\frac{1}{2} \Rightarrow [x - \frac{1}{x}] \geq 0 \rightarrow x - \frac{1}{x} \geq 0$
 $\Rightarrow x \geq \frac{1}{x} \quad D_f = [\frac{1}{x}, +\infty)$
 (✓) ✓

ب) $y = \sqrt{[x - \frac{1}{x}] + [-x + \frac{1}{x}]}$
 $[x - \frac{1}{x}] + [-x + \frac{1}{x}] \geq 0 \rightarrow x \in \mathbb{Z} \quad [x] + [-x] = 0 \checkmark$
 $\rightarrow x \notin \mathbb{Z} \quad [x] + [-x] = -1 \times$
 $x - \frac{1}{x} \in \mathbb{Z} \rightarrow x \in \mathbb{Z} + \frac{1}{x}$
 $D_f = \{x \mid x = k + \frac{1}{x}, k \in \mathbb{Z}\}$

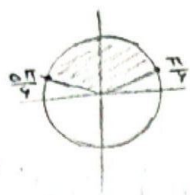
الف) $y = \frac{x}{\sin x - 1}$
 $\sin x - 1 \neq 0 \rightarrow \sin x \neq 1 (90^\circ)$
 $D_f = \mathbb{R} - \{k\pi + \frac{\pi}{2}\}$
 (✓) ✓

ب) $y = \frac{x}{x \sin^2 x - 1}$
 $x \sin^2 x - 1 \neq 0 \rightarrow \sin x \neq \pm \frac{\sqrt{x}}{x}$
 $D_f = \mathbb{R} - \{\frac{k\pi}{x} + \frac{\pi}{2}\}$

$$\text{الف) } y = \sqrt{x \sin x - 1}$$

$$x \sin x - 1 \geq 0 \rightarrow \sin x \geq \frac{1}{x}$$

$$D_f = \left[2k\pi + \frac{\pi}{4}, 2k\pi + \frac{5\pi}{4} \right]$$



Ⓟ ✓

$$\text{ب) } y = \sqrt{\cos x - 1}$$

$$\cos x - 1 \geq 0 \rightarrow \cos x \geq 1$$

$$D_f = \{2k\pi\}$$



$$\text{الف) } \cos y = |x| - 1$$

$$-1 \leq |x| - 1 \leq 1 \xrightarrow{+1} -1 \leq |x| \leq 2 \rightarrow -2 \leq x \leq 2$$

$$D_f = [-2, -1] \cup [1, 2]$$

Ⓟ ✓

$$\text{ب) } y = \arcsin(x^2 + x + 1)$$

$$x^2 + x + 1 \geq -1 \rightarrow x^2 + x + 2 \geq 0 \rightarrow (x+1)(x+2) \geq 0 \quad ①$$

$$x^2 + x + 1 \leq 1 \rightarrow x^2 + x \leq 0 \rightarrow x(x+1) \leq 0 \quad ②$$

$$\begin{array}{c} \text{①} \quad \begin{array}{c} -2 \quad -1 \\ + \quad - \quad + \end{array} \quad \text{②} \quad \begin{array}{c} -1 \quad 0 \\ + \quad - \quad + \end{array} \quad D_f = ① \cap ② \\ (-\infty, -2] \cup [-1, +\infty) \quad [-1, 0] \end{array}$$

$$D_f = [-2, -1] \cup [-1, 0]$$

$$\text{الف) } \sqrt{x - \log_r(x-1)} = y$$

$$x - \log_r(x-1) \geq 0 \rightarrow \log_r(x-1) \leq x \rightarrow x-1 \leq r^x \rightarrow$$

$$x \leq 11 \rightarrow D_f = (r, 11]$$

$$x-1 > 0 \rightarrow x > 1$$

Ⓟ ✓

$$\text{ب) } y = \log(x^2 - x)$$

$$x^2 - x > 0 \rightarrow x(x-1)(x+1) > 0$$



$$D_f = (-1, 0) \cup (1, +\infty)$$

$$\text{الف) } y = \frac{x}{x^2 + 1}$$

$$D_f = \mathbb{R}$$

$$x^2 + 1 \neq 0 \rightarrow x^2 \neq -1 \quad \text{ممنوع}$$

$$\text{ج) } y = \frac{x}{x^2 - r}$$

$$D_f = \mathbb{R} - \{\log_r r\}$$

$$x^2 - r \neq 0 \rightarrow x^2 \neq r(\log_r r)$$

Ⓟ ✓

$$\text{ب) } y = \frac{x}{x^2 - 1}$$

$$D_f = \mathbb{R} - \{0\}$$

$$x^2 - 1 \neq 0 \rightarrow x^2 \neq 1 \rightarrow x \neq 0$$

$$\text{د) } y = \frac{x}{x^2 - r}$$

$$D_f = \mathbb{R} - \{\log_r r\}$$

$$x^2 - r \neq 0 \rightarrow x^2 \neq r(\log_r r)$$

$$\text{الف) } (x+1)!$$

$$x+1 \in \mathbb{N} \rightarrow x \in \mathbb{N} - 1 \rightarrow x \in \frac{\mathbb{N}-1}{r}$$

$$D_f = \left\{ x \mid x = \frac{k-1}{r}, k \in \mathbb{N} \right\}$$

Ⓟ ✓

$$\text{ب) } \left(\frac{rx-1}{rx-a} \right)!$$

$$\frac{rx-1}{rx-a} \in \mathbb{N} \rightarrow rx-1 = rxw - aw$$

$$\rightarrow rx - rxw = 1 - aw \rightarrow x = \frac{1-aw}{r-rw}$$

$$D_f = \left\{ x \mid x = \frac{1-ak}{r-rk}, k \in \mathbb{N} \right\}$$