

الف) $y = \frac{x^2 - 4x}{x^3 - x} \geq 0 \rightarrow x^3 - x \neq 0 \rightarrow \begin{cases} x^2 - 4x = x(x-4) \\ x^3 - x = x(x^2 - 1) = x(x+1)(x-1) \end{cases}$

ب) $y = \frac{x^6 - x^3}{x^3 - x} \geq 0 \rightarrow x^3 - x \neq 0 \rightarrow \begin{cases} x^6 - x^3 = x^3(x^3 - 1) = x^3(x-1)(x^2+x+1) \\ x^3 - x = x(x^2 - 1) = x(x+1)(x-1) \end{cases}$

ج) $y = x + |x| > 0 \rightarrow |x| < x$ نتیجه سری $D_f = (0, +\infty) \cup \mathbb{R}^+$

د) $y = \frac{1}{\sqrt{x|x|}} \rightarrow x|x| > 0 \rightarrow D_f = (0, +\infty)$

الف) $y = 2 - [x] \geq 0 \rightarrow [x] \leq 2 \rightarrow x < 3 \rightarrow D_f = (-\infty, 3)$

ب) $y = 2 - [x] > 0 \rightarrow [x] < 2 \rightarrow x < 2 \rightarrow D_f = (-\infty, 2)$

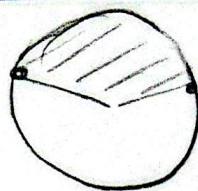
الف) $y = \sqrt{\left[x - \frac{1}{3}\right]\left[x + \frac{2}{3}\right]} \rightarrow \left[x - \frac{1}{3}\right]\left[x + \frac{2}{3}\right] \geq 0$

ب) $y = \sqrt{\left[x - \frac{1}{3}\right]\left[-x + \frac{1}{3}\right]} \rightarrow \left[x - \frac{1}{3}\right]\left[-x + \frac{1}{3}\right] \geq 0$

الف) $y = \frac{2}{\sin x - 1}$ $\sin x - 1 \neq 0$ $\sin x \neq 1$ $D_f = \mathbb{R} - \left\{2k\pi + \frac{\pi}{2}\right\}$

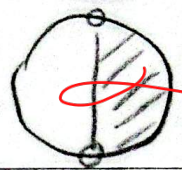
ب) $y = \frac{2}{2\sin^2 x - 1}$ $2\sin^2 x - 1 \neq 0$ $2\sin^2 x \neq 1$ $\sin^2 x \neq \frac{1}{2}$ $\sin x \neq \pm \sqrt{\frac{1}{2}}$ $D_f = \mathbb{R} - \left\{\frac{k\pi}{2} + \frac{\pi}{4}, \frac{k\pi}{2} + \frac{3\pi}{4}\right\}$

الف) $y = \sqrt{r \sin x - 1}$ $r \sin x - 1 \geq 0$
 $r \sin x \geq 1$
 $\sin x \geq \frac{1}{r}$



$D_f = [\frac{\pi}{4} + 2k\pi, \frac{3\pi}{4} + 2k\pi]$

ب) $y = \sqrt{\cos x - 1}$ $\cos x - 1 \geq 0$
 $\cos x \geq 1$
 $\cos x = 1$
 $x = 2k\pi$



$D_f = \{2k\pi\}$

الف) $y = \sqrt{r - \log_p(x-r)}$ $r - \log_p(x-r) \geq 0$
 $-\log_p(x-r) \geq -r$
 $\log_p(x-r) \leq r$
 $x-r \leq p^r$
 $x \leq r + p^r$
 $x-r > 0$
 $x > r$

✓

ب) $y = \log(x^3 - x)$

$x^3 - x > 0 \rightarrow x(x^2 - 1) > 0$
 $x(x+1)(x-1) > 0$

$D_f = (-1, 0) \cup (1, +\infty)$

الف) $y = \frac{r}{r^x + 1}$ $r^x + 1 \neq 0$
 $r^x \neq -1$
 $D_f = \mathbb{R}$

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ب) $y = \frac{r}{r^x - 1}$ $r^x - 1 \neq 0$
 $r^x \neq 1$
 $x \neq 0$
 $D_f = \mathbb{R} \setminus \{0\}$

ج) $y = \frac{r}{r^x - r}$ $r^x - r \neq 0$
 $r^x \neq r$
 $x \neq \log_r r$
 $x \neq 1$
 $D_f = \mathbb{R} \setminus \{1\}$

د) $y = \frac{r}{r^x - r^r}$ $r^x - r^r \neq 0$
 $r^x \neq r^r$
 $x \neq r$
 $D_f = \mathbb{R} \setminus \{r\}$

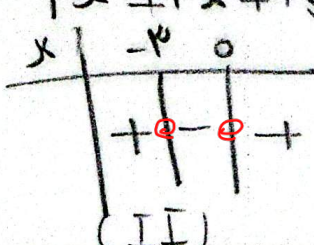
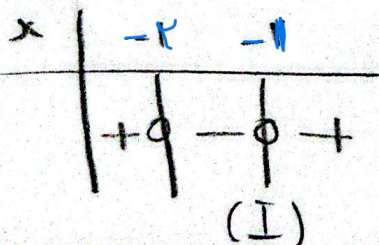
الف) $y = (r^x + 1)!$ $r^x + 1 \in \mathbb{N} \rightarrow r^x \in \mathbb{N} - 1$
 $D_f = \{x \mid x = \frac{k-1}{r}, k \in \mathbb{N}\}$

ب) $y = \left(\frac{r^x - r}{r^x - r^r}\right)!$ $\frac{r^x - r}{r^x - r^r} \in \mathbb{N}$
 $D_f = \{x \mid x = \frac{\omega k - r}{rk - r}, k \in \mathbb{N}\}$

الف) $\cos y = |x| - r$ $-1 \leq |x| - r \leq 1$
 $|x| \leq r$
 $-r \leq x \leq r$
 $|x| \geq r \rightarrow x \leq -r \vee x \geq r$
 $D_f = [-r, r] \cup [r, r]$

ب) $y = \arcsin(x^r + r^r x + 1)$

$-1 \leq x^r + r^r x + 1 \leq 1$
 $x^r + r^r x + 1 \geq -1 \rightarrow x^r + r^r x + 2 \geq 0 \rightarrow (x+1)(x^r + r^r) \geq 0$
 $x^r + r^r x + 1 \leq 1 \rightarrow x^r + r^r x \leq 0 \rightarrow x(x + r^r) \leq 0$



$D_f = (I) \cap (II) = [-r^r, -1]$
 $\cup [0, +\infty)$

ك - الف

$$\left\lceil n - \frac{1}{r} \right\rceil + \left\lceil n - \frac{1}{r} \right\rceil + 1 \geq 0 \rightarrow 2 \left\lceil n - \frac{1}{r} \right\rceil \geq -1 \rightarrow \left\lceil n - \frac{1}{r} \right\rceil \geq -\frac{1}{2} \rightarrow n - \frac{1}{r} \geq 0 \rightarrow n \geq \frac{1}{r}$$

$$n - \frac{1}{r} \in \mathbb{Z} \rightarrow n \in \mathbb{Z} + \frac{1}{r}$$

$$D_y = \{n \mid n = k + \frac{1}{r}, k \in \mathbb{Z}\}$$

$$\begin{aligned} [n] + [-n] &\rightarrow \in \mathbb{Z} = 0 \\ &\rightarrow \notin \mathbb{Z} = -1 \end{aligned}$$

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