

سؤال ١١

$$y = \sqrt{a f(a)} \Rightarrow a f(a) \geq 0$$

$$\begin{aligned} [-\infty, -\infty) &\Rightarrow f(a) > 0, a < 0 \Rightarrow a \times f(a) = \text{منفي} \\ [-\infty, 0] &\Rightarrow f(a) \leq 0, a \leq 0 \Rightarrow a \times f(a) = \text{منفي} \\ (0, 2) &\Rightarrow a > 0, f(a) < 0 \Rightarrow a \times f(a) = \text{منفي} \\ [2, \infty) &\Rightarrow a > 0, f(a) \geq 0 \Rightarrow a \times f(a) = \text{موجب} \end{aligned}$$

$$\Rightarrow D_f = [-\infty, 0] \cup [2, \infty)$$

سؤال ١٢

$$y = \sqrt{\frac{-a}{f(a)}} \Rightarrow \frac{-a}{f(a)} \geq 0 \Rightarrow f(a) \neq 0 \Rightarrow a = -3, 0, 2$$

$$D_y = (-3, 0) \cup (0, 2) \Rightarrow \boxed{-2, -1, 1, 2} \text{ اعداد صحيحة}$$

سؤال ١٣

$$f(a) - 2f(2) = a^2 - 3a + 4$$

$$f(2) - 2f(2) = 4 - 6 + 4 \Rightarrow -f(2) = 2 \Rightarrow -2 = f(2)$$

$$f(-2) - 2f(-2) = (-2)^2 - 3(-2) + 4 \Rightarrow f(-2) + 4 = 4 + 6 + 4 \Rightarrow f(-2) + 4 = 14 \Rightarrow \boxed{f(-2) = 10}$$

سؤال ١٤

$$f(a) = \begin{cases} a - \sqrt{a+4} & a > 3 \Rightarrow 4 \\ 2a+3 & a \leq 3 \Rightarrow 1, 2 \end{cases}$$

$$f(1) = 1 \times 1 + 3 = 4 \Rightarrow f(f(1)) = f(4) = 2$$

$$f(4) = 4 - \sqrt{4} = 4 - 2 = 2$$

$$f(f(4)) = f(2) = 2 \times 2 + 3 = 7$$

$$\Rightarrow \boxed{\frac{f(f(4))}{7} + \frac{f(f(1))}{2} = 9}$$

سوال (5)

$$f(a) = a^2 - b a + 2 \Rightarrow f(a-1) = a(a-1)^2 - b(a-1) + 2 \Rightarrow a^2 - 2a + a - b a + b + 2$$

$$f(a-1) - f(a) = 4a + 2$$

$$\Rightarrow f(a-1) - f(a) = (-2a)a + a + b$$

$$(-2a)a + a + b = 4a + 2 \Rightarrow -2a = 4 \Rightarrow a = -2$$

$$a - b = -3 - 5 = -8 \Rightarrow -2a = 4 \Rightarrow a = -2, a + b = 2 \Rightarrow -2 + b = 2 \Rightarrow b = 4$$

سوال (6)

$$f(a) = a^2 - 3a + 1$$

$$f(3) = 9 - 9 + 1 = 1 \Rightarrow f(3) = 1, f(-1) = 1 + 3 + 1 = 5 \Rightarrow f(-1) = 5$$

$$f(2) = 4 - 6 + 1 = -1 \Rightarrow f(2) = -1$$

$$\Rightarrow \frac{2f(3) + f(-1)}{f^2(2)} = \frac{2 \times 1 + 5}{(-1)^2} = \frac{7}{1} = 7$$

سوال (7)

$$f(a) = \begin{cases} 3a + 1 & a \geq 2 \\ a^2 & a < 2 \end{cases} \Rightarrow f(2) = 7, f(-2) = 4$$

$$f(f) = 3 \times f + 1 = 13, f(2) = 3 \times 2 + 1 = 7, f(-2) = 1 - 2^2 = -3$$

$$\frac{3f(f) - 2f(-2)}{f(2)} = \frac{3 \times 13 - 2 \times (-3)}{7} = \frac{45}{7}$$

سوال (8)

$$P = \{(2, 4), (3, 2), (4, 1), (1, 4)\}$$

$$P(f) = 1, f(3) = 2, f(1) = 4$$

$$P(P(f)) = 4, P(f(f)) = 4$$

$$\Rightarrow \frac{P(P(f)) - P(f(f))}{P(1) - 2} = \frac{4 - 4}{4 - 2} = \frac{0}{2} = 0$$

سؤال ٩

$$f(a) = \frac{a^2 + 4a + 9}{a^2 + 4a + 5}$$

$$f(\sqrt{3}-2) = \frac{(\sqrt{3}-2)^2 + 4(\sqrt{3}-2) + 9}{(\sqrt{3}-2)^2 + 4(\sqrt{3}-2) + 5} = \frac{-1+9}{-1+5} = \frac{8}{4} = 2$$

$$f\left(\frac{f(\sqrt{3}-2)}{2}\right) = \frac{2^2 + 4 \times 2 + 9}{2^2 + 4 \times 2 + 5} = \frac{4+8+9}{4+8+5} = \frac{21}{17}$$

$$f(a) = a^2 + \frac{1}{a^2} \Rightarrow f(\sqrt{a}) = a + \frac{1}{a}$$

$$f(\sqrt{a}) - f(a) = \left(a + \frac{1}{a}\right) - \left(a^2 + \frac{1}{a^2}\right) = a + \frac{1}{a} + 2\frac{a}{a} - a^2 - \frac{1}{a^2} = 2$$

سؤال ١٥