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نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ...۱۰... کلاس .....

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الف) $t_n = 3n^2 - 1 \Rightarrow$ $n=1 \to 2$, $n=2 \to 11$, $n=3 \to 26$, $n=4 \to 47$, $n=5 \to 74$ $\Rightarrow 2, 11, 26, 47, 74, \dots$ (۳)

ب) $t_n = \frac{2n+1}{n+4} \to$ $n=1 \to \frac{3}{5}$, $n=2 \to \frac{5}{6}$, $n=3 \to \frac{7}{7}=1$, $n=4 \to \frac{9}{8}$, $n=5 \to \frac{11}{9}$ $\frac{3}{5}, \frac{5}{6}, 1, \frac{9}{8}, \frac{11}{9}, \dots$

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الف) $\frac{(-1)^n}{\sqrt{n+3}} \overset{n=13}{\Rightarrow} \frac{-1}{4} \Rightarrow -\frac{1}{4}$ (۲)

ب) $t_n = 2n - \left[\frac{n}{4}\right] \Rightarrow 2\times 13 - \underbrace{\left[\frac{13}{4}\right]}_{3} \Rightarrow 26 - 3 = 23$

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الف) $t_n = 2n - 10$ $\quad 2n-10 < 0 \to 2n < 10 \to n < 5 \to 1, 2, 3, 4 \to$ ۴ جمله (۲)

ب) $t_n = n^2 - 14n + 45$ $\quad n^2 - 14n + 45 < 0 \to (n-10)(n-4) < 0$

$\begin{cases} n-10 > 0,\ n-4 < 0 \to n > 10,\ n < 4 \to \cap \to \emptyset \\ n-10 < 0,\ n-4 > 0 \to n < 10,\ n > 4 \to \cap \to (4, 10) \to 5, 6, 7, 8, 9 \to \text{۵ جمله} \end{cases}$

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الف) $t_n = 2n - 14 \to 2n - 14 \le 0 \to 2n \le 14 \to n \le 7 \to 1, 2, 3, 4, 5, 6, 7 \to$ ۷ جمله

ب) $t_n = n^2 - 12n + 27 \to (n-3)(n-9) \le 0$

$\left.\begin{cases} n-3 \ge 0,\ n-9 \le 0 \to n \ge 3,\ n \le 9 \to [3, 9] \\ n-3 < 0,\ n-9 > 0 \to n < 3,\ n > 9 \to \emptyset \\ n = 3,\ n = 9 \end{cases}\right\} \to [3, 9] \to 3, 4, 5, 6, 7, 8, 9$ (۲)

۷ جمله

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$\begin{cases} a_4 = a_1 + 3d = 7 \\ a_7 = a_1 + 6d = 22 \end{cases} \to 3d = 15 \to d = 5$ (۵)

$a_1 + 6 = 7 \to a_1 = -2$

$a_3 = a_1 + 2d \Rightarrow -2 + 2(5) =$ ۸

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الف) $t_n = n^2 - 9n + 2 \rightarrow a > 0 \rightarrow \min \quad x_s = \frac{-b}{2a} \Rightarrow \frac{-(-9)}{2\times 1} = 4$

$y_s = 4^2 - 9(4) + 2 = \boxed{-7}$ ✓

ب) $t_n = \underbrace{n^2 + 4n + 1}_{\text{مثبت}}$ مثبت $\rightarrow$ ✓ $\Rightarrow n = 1 \Rightarrow 1 + 4 + 1 = \boxed{6}$

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$t_n = -4, -7, -8, -7, -4, \ldots$ $\Rightarrow 2a = 2 \rightarrow a = 1$

$-3, -1, +1, +3$

$+2, +2, +2$

$an^2 + bn + c \Rightarrow \begin{cases} n=1 \Rightarrow 1 + b + c = -4 \\ n=4 \Rightarrow 4 + 2b + c = -7 \end{cases} \Rightarrow -3 - b = 3 \Rightarrow \xi, b = -4$

$n=1 \Rightarrow 1 - 4 + c = 4 \Rightarrow c = 1$

(جواب الف) $a_n = n^2 - 4n + 1 \rightarrow \underbrace{(10)^2}_{100} - \underbrace{4(10)}_{40} + 1 = \boxed{61}$ ✓

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$t_n = -4, -1, 2, 7, \ldots$ $\quad 2a = 4 \rightarrow a = 2$

$+a, +a, +a$

$+2, +2$

$an^2 + bn + c \Rightarrow \begin{cases} n=1 \Rightarrow 2(1)^2 + 1(b) + c = -4 \\ n=2 \Rightarrow \underbrace{2(2)^2}_{8} + \underbrace{2(b)}_{2b} + c = -1 \end{cases} \rightarrow 4 + b = 0 \Rightarrow \xi, b = -1$

$n=1 \rightarrow 2(1)^2 - 1 + c = -4 \rightarrow c = -7$

(جواب کلی) $a_n = \boxed{2n^2 - n - 7}$ ✓

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$(-\infty, a+4] \cap [2a+1, +\infty)$ $\rightarrow$ تناهی

$a + 4 \not< 2a + 1 \rightarrow \boxed{3 \not< a}$ یا $\boxed{[3, +\infty)}$ ✓

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الف) $3, 4, 9, \ldots, 99 \Rightarrow \frac{a_n - a_1}{d} + 1 \Rightarrow \frac{99 - 3}{3} + 1 = \boxed{33}$ تا $\rightarrow n(A) = 33$ ✓

ب) $0, 10, 15, \ldots, 95 \Rightarrow \frac{a_n - a_1}{d} + 1 \Rightarrow \frac{100 - 0}{5} + 1 = \boxed{20}$ تا $\Rightarrow n(B) = 20$ ✓

ج) $15, 30, 45, \ldots, 90 \Rightarrow \frac{a_n - a_1}{d} + 1 \Rightarrow \frac{90 - 15}{15} + 1 = \boxed{6}$ تا $\rightarrow n(A \cap B) = 6$ ✓

د) $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 33 + 20 - 6 = \boxed{47}$ تا $\rightarrow n(A \cup B) = 47$ ✓