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مهسا انامداغتی - دهم دختر C - تکلیف شماره ۲۰

Date: Subject:

الف) $(\sin 135^\circ + \sin 300^\circ)(\cos 315^\circ - \cos 150^\circ)$ ①

$\sin 135^\circ = \sin(90^\circ + 45^\circ) = \sin 90^\circ \cos 45^\circ + \sin 45^\circ \cos 90^\circ =$

$1 \times \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \times 0 = \frac{\sqrt{2}}{2} + 0 = \frac{\sqrt{2}}{2}$

$\sin 300^\circ = \sin(150^\circ + 150^\circ) = 2\sin 150^\circ \times \cos 150^\circ = 2 \times \frac{1}{2} \times (-\frac{\sqrt{3}}{2}) = -\frac{\sqrt{3}}{2}$

$\cos 315 = \cos(270^\circ + 45^\circ) = \cos 270^\circ \times \cos 45^\circ - \sin 270^\circ \times \sin 45^\circ =$

$0 \times \frac{\sqrt{2}}{2} - (-1) \times \frac{\sqrt{2}}{2} = 0 + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$

$\cos 150^\circ = \cos(90 + 60) = \cos 90 \cos 60 - \sin 90 \sin 60 =$

$0 \times \frac{1}{2} - 1 \times \frac{\sqrt{3}}{2} = 0 - \frac{\sqrt{3}}{2}$

$(\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2})(\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}) = (\frac{\sqrt{2}}{2})^2 - (\frac{\sqrt{3}}{2})^2 = \frac{2}{4} - \frac{3}{4} = \boxed{-\frac{1}{4}}$

ب) $\frac{1 + 3\tan^2 210}{4\cos^2 330 + 2\cos^2 225}$

$3\tan^2 210^\circ = 3(\frac{1-\cos 2\alpha}{1+\cos 2\alpha}) = 3(\frac{1-\cos 420}{1+\cos 420}) = 3(\frac{1-\cos 60^\circ}{1+\cos 60}) = 3(\frac{1-\frac{1}{2}}{1+\frac{1}{2}})$

$3(\frac{\frac{1}{2}}{\frac{3}{2}}) = 3 \times \frac{1}{3} = 1$

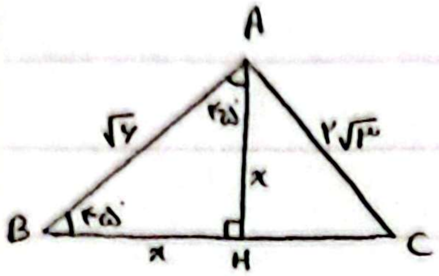
$4\cos^2 330 = 4(\frac{1+\cos 2\alpha}{2}) = 2(1+\cos 660) = 2(1+\cos 300) = 2(1+\frac{1}{2})$

$= 2(\frac{3}{2}) = 3$

$2\cos^2 225 = 2(\frac{1+\cos 2\alpha}{2}) = 1+\cos 2\alpha = 1+\cos 450 = 1+\cos 90 = 1+0 = 1$

$\frac{1+1}{3+1} = \frac{2}{4} = \boxed{\frac{1}{2}}$

② الف)



$$x^2 + x^2 = (\sqrt{6})^2$$

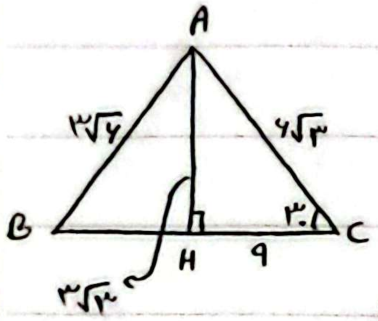
$$2x^2 = 6 \rightarrow x^2 = 3 \rightarrow x = \pm\sqrt{3} \rightarrow x = +\sqrt{3}$$

مثبت را انتخاب می کنیم چون طول یک ضلع نمی تواند منفی باشد

$$AH = \sqrt{3} \quad (2\sqrt{3})^2 - (\sqrt{3})^2 = 4\times 3 - 3 = 12 - 3 = 9 \rightarrow \sqrt{9} = 3$$

$$\boxed{HC = 3}$$

ب)



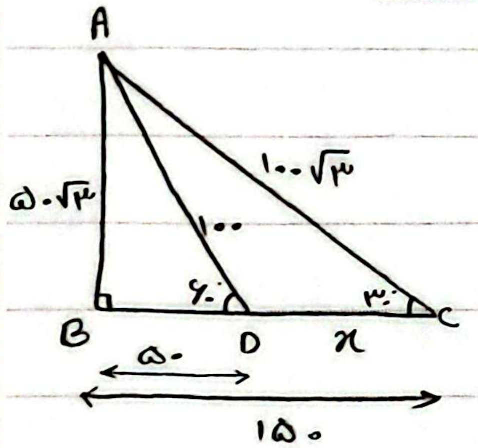
$$\cos 30^\circ = \frac{CH}{AC} = \frac{9}{AC} = \frac{\sqrt{3}}{2} \rightarrow AC = \frac{18}{\sqrt{3}} = \frac{18\sqrt{3}}{3} = 6\sqrt{3}$$

$$(6\sqrt{3})^2 - 9^2 = 108 - 81 = 27 \rightarrow \sqrt{27} = 3\sqrt{3}$$

$$\sin B = \frac{AH}{AB} = \frac{3\sqrt{3}}{3\sqrt{6}} = \frac{\sqrt{3}}{\sqrt{6}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \rightarrow \sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$\boxed{\hat{B} = 45^\circ}$$

③ الف)



$$\sin 30^\circ = \frac{AB}{AC} = \frac{50\sqrt{3}}{AC} = \frac{1}{2} \rightarrow AC = 100\sqrt{3}$$

$$(100\sqrt{3})^2 - (50\sqrt{3})^2 = 30000 - 7500 = 22500$$

$$\rightarrow \sqrt{22500} = \sqrt{150^2} = 150$$

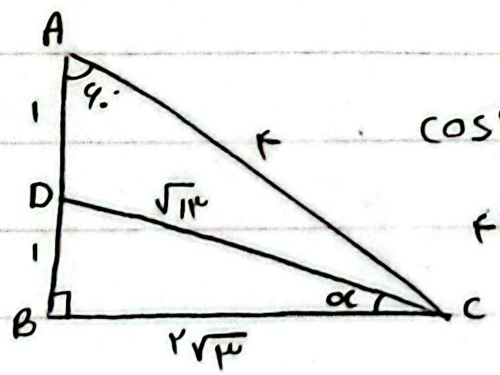
$$\sin 60 = \frac{AB}{AD} = \frac{50\sqrt{3}}{AD} = \frac{\sqrt{3}}{2} \rightarrow AD = \frac{100\sqrt{3}}{\sqrt{3}} = 100$$

$$100^2 - (50\sqrt{3})^2 = 10000 - 7500 = 2500$$

$$\rightarrow \sqrt{2500} = \sqrt{50^2} = 50$$

$$x = 150 - 50 = 100 \rightarrow \boxed{x = 100}$$

ب)



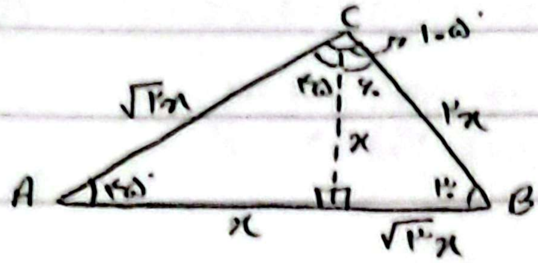
$$\cos 60^\circ = \frac{AB}{AC} = \frac{2}{AC} = \frac{1}{2} \rightarrow AC = 4$$

$$4^2 - 2^2 = 16 - 4 = 12 \rightarrow \sqrt{12} = \sqrt{4\times 3} = 2\sqrt{3}$$

$$1^2 + (2\sqrt{3})^2 = 1 + 12 = 13 \rightarrow \sqrt{13}$$

$$\sin\alpha = \frac{1}{\sqrt{13}} \rightarrow \sin\alpha = \boxed{\frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}}$$

۴)



$(2x)^2 - x^2 = 4x^2 - x^2 = 3x^2$

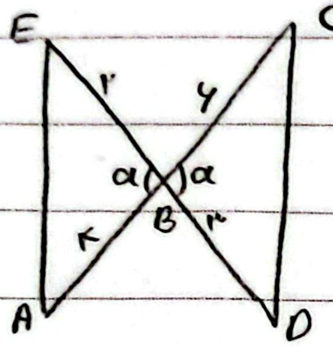
$\rightsquigarrow \sqrt{3x^2} = \sqrt{3}\,x$

$x^2 + x^2 = 2x^2 \rightsquigarrow \sqrt{2x^2} = \sqrt{2}\,x$

$$\frac{AB}{AC} = \frac{x+\sqrt{3}x}{\sqrt{2}x} = \frac{x(1+\sqrt{3})}{x\sqrt{2}} = \frac{1+\sqrt{3}}{\sqrt{2}}$$

$$\rightsquigarrow \boxed{\frac{AB}{AC} = \frac{1+\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{2}}$$

۵)

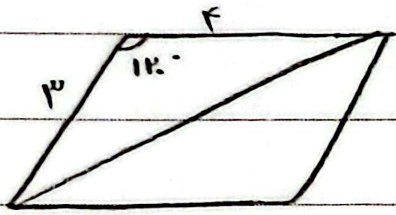


$S_{\hat{ABE}} = \frac{1}{2} \times 2 \times 4 \times \sin\alpha = 4\sin\alpha$

$S_{\hat{BCD}} = \frac{1}{2} \times 6 \times 3 \times \sin\alpha = 9\sin\alpha$

$$\frac{4\sin\alpha}{9\sin\alpha} = \boxed{\frac{4}{9}}$$

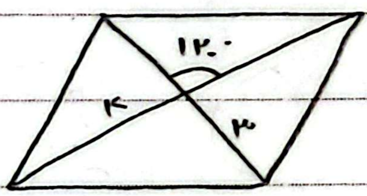
۶) الف)



$\frac{1}{2} \times 3 \times 4 \times \sin 120^\circ = \frac{12}{2}\sin 120^\circ = 6 \times \frac{\sqrt{3}}{2}$

$= 3\sqrt{3} \times 2 = \boxed{6\sqrt{3}}$

ب)



$\frac{1}{2} \times 3 \times 4 \times \sin 120 = 6 \times \sin 120 = 6 \times \frac{\sqrt{3}}{2}$

$= \boxed{3\sqrt{3}}$

(۴،۳ طول نصف قطرها است)

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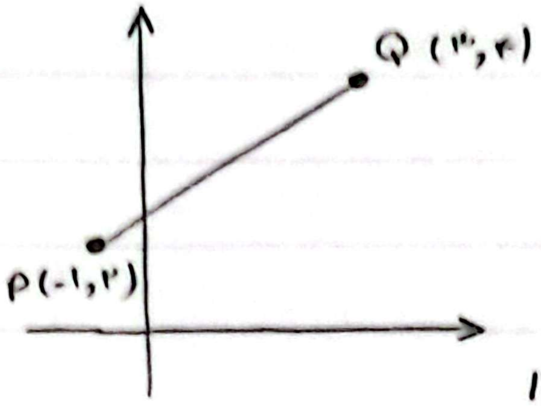

$$\text{شیب} = \frac{\Delta y}{\Delta x} = \frac{4-2}{3-(-1)} = \frac{2}{3+1} = \frac{2}{4} = \frac{1}{2}$$

$$\rightarrow \tan\theta = \frac{1}{2}$$

$$1+\tan^2\theta = \frac{1}{\cos^2\theta} \rightarrow 1+\frac{1}{4} = \frac{1}{\cos^2\theta} = \frac{5}{4}$$

$$\rightarrow \cos^2\theta = \frac{4}{5} \rightarrow \cos\theta = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\boxed{\cos\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}}$$

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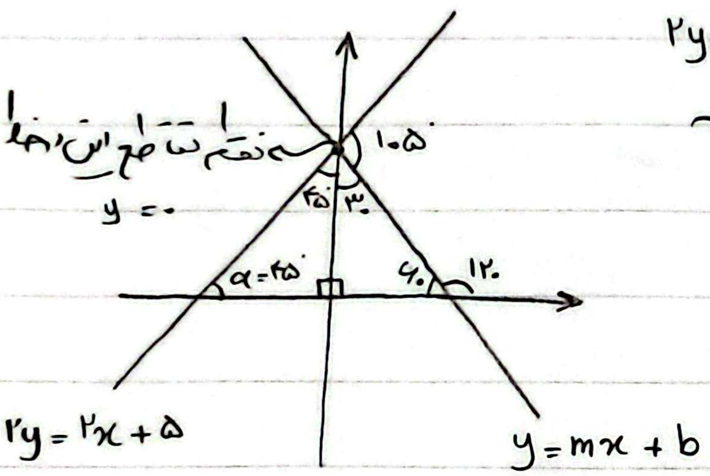

$$2y = 2x+5 \rightarrow y = x + 2{,}5$$

$$\rightarrow \tan\alpha = 1 \rightarrow \alpha = 45^\circ$$

$$\tan 120^\circ = \frac{\sin 120^\circ}{\cos 120^\circ} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$

$$\text{ل}\ 120^\circ = \left| \begin{matrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{matrix} \right.$$

$$\rightarrow \boxed{m = -\sqrt{3}}$$

$$y = x + 2{,}5 \rightarrow y = 0 + 2{,}5 \rightarrow y = \frac{5}{2}$$

$$y = -\sqrt{3}x + b \rightarrow \frac{5}{2} = -\sqrt{3}\times 0 + b \rightarrow \boxed{b = \frac{5}{2}}$$

$$\boxed{mb = -\frac{5\sqrt{3}}{2} = -2{,}5\sqrt{3}}$$

۲ 9

$f(x) = \cos(\pi + x) + \sin(\frac{\pi}{2} - x) - \tan(\frac{3\pi}{2} + x)$

$\cos(\pi + x) = -\cos x$

$\sin(\frac{\pi}{2} - x) = \cos x \quad \rightarrow \cancel{-\cos x + \cos x} + \cot x = \cot x$

$\tan(\frac{3\pi}{2} + x) = -\cot x \quad f(x) = \cot x$

$f(\frac{5\pi}{6}) = f(150^\circ) = \cot 150^\circ = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

$\rightarrow \cot 150^\circ = \boxed{-\sqrt{3}}$

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۵ 10

$$\frac{\sin(\frac{17\pi}{2} + x) + \cos(\frac{15\pi}{2} - x)}{\cos(3\pi + x) - \sin(6\pi - x)}$$

$\sin(\frac{17\pi}{2} + x) = \sin\frac{17\pi}{2}\cos x + \cos\frac{17\pi}{2}\sin x$

$\cos(\frac{17\pi}{2}) = \cos(\frac{16\pi}{2} + \frac{\pi}{2}) = \cos(8\pi + \frac{\pi}{2}) = \cos\frac{\pi}{2} = \cos 90^\circ = 0$

$\sin(\frac{17\pi}{2}) = \sin(\frac{16\pi}{2} + \frac{\pi}{2}) = \sin(8\pi + \frac{\pi}{2}) = \sin\frac{\pi}{2} = \sin 90^\circ = 1$

$\rightarrow 1 \times \cos x + 0 \times \sin x = \boxed{\cos x}$

$\cos(3\pi + x) = \cos(\pi + x) = \boxed{-\cos x}$

$\sin(6\pi - x) = \sin(2\pi - x) = \boxed{-\sin x}$

$\cos(\frac{15\pi}{2} - x) = \cos\frac{15\pi}{2}\cos x + \sin\frac{15\pi}{2}\sin x$

$\cos\frac{15\pi}{2} = \cos(\frac{12\pi}{2} + \frac{3\pi}{2}) = \cos(6\pi + \frac{3\pi}{2}) = \cos\frac{3\pi}{2} = 0$

$\sin\frac{15\pi}{2} = \sin(\frac{12\pi}{2} + \frac{3\pi}{2}) = \sin(6\pi + \frac{3\pi}{2}) = \sin\frac{3\pi}{2} = -1$

$\rightarrow 0 \times \cos x + (-1) \times \sin x = 0 + \sin x = \boxed{-\sin x}$

$$\frac{\cos x - \sin x}{-\cos x - (-\sin x)} = \frac{\cos x - \sin x}{-\cos x + \sin x} = \frac{\cos x - \sin x}{-1(\cos x - \sin x)} = \boxed{-1}$$

جواب نهایی.