پریسان دهقانی

۱- الف) $(\sin 135 + \sin 300)(\cos 315 - \cos 150)$ $\rightarrow \sin 135 = \frac{\sqrt{2}}{2}$ / $\cos 150 = -\frac{\sqrt{3}}{2}$

$\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\right) = 2\left(-\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$ $\cos 315 = \frac{\sqrt{2}}{2}$ / $\sin 300 = -\frac{\sqrt{3}}{2}$

$= \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

ب) $\frac{1 + 3\tan^2 210}{4\cos^2 330 + \cos^2 225} = ?$ $\quad 3\tan^2 210 = 3\left(\frac{1-\cos 60}{1+\cos 60}\right) = 3\left(\frac{1-\frac{1}{2}}{1+\frac{1}{2}}\right) = 3\left(\frac{1}{3}\right) = 1$

$\rightarrow \frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2 \times 4 - \frac{1}{2}} = \frac{1+1}{3+1} = \frac{1}{2}$

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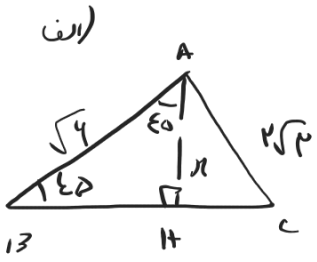
۲- الف)



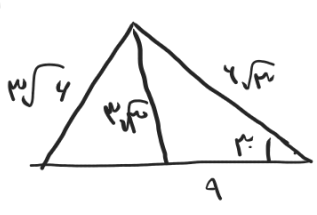

$x^2 + x^2 = (\sqrt{4})^2$

$2x^2 = x^2 = 3 \rightarrow x = \sqrt{3}$

$AH = \sqrt{3}$ $\quad (2\sqrt{3})^2 - (\sqrt{3})^2 = 6 \times 3 - 3 = 12 - 3 = 9 \rightarrow \sqrt{9} = 3$

$HC = 9$

ب)




$\cos 30^\circ = \frac{CH}{AC} = \frac{9}{AC} = \frac{\sqrt{3}}{2} \rightarrow AC = \frac{18}{\sqrt{3}} = \frac{18\sqrt{3}}{3} = 6\sqrt{3}$

$(4\sqrt{3})^2 - 9^2 = 108 - 81 = 27 \rightarrow \sqrt{27} = 3\sqrt{3}$

$\sin B = \frac{AH}{AB} = \frac{3\sqrt{3}}{3\sqrt{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2} \rightarrow \sin 60$

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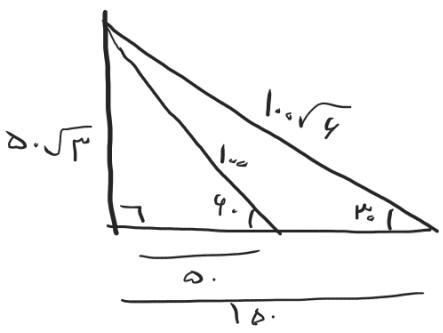
۳- الف) $\sin 30 = \frac{AB}{AC} = \frac{50\sqrt{3}}{AC} = \frac{1}{2} \rightarrow AC = 100\sqrt{3}$

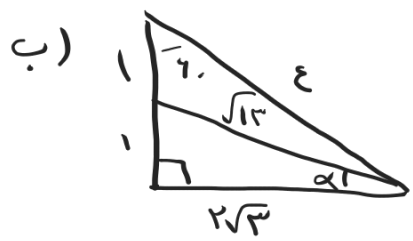



$(100\sqrt{3})^2 - (50\sqrt{3})^2 = 30000 - 7500 = 22500$

$\sqrt{22500} = \sqrt{150} = 150$ $\quad x = 150 - 50 = 100$

$\cos 60 = \frac{AB}{AC} = \frac{2}{4} = \frac{1}{2} \Rightarrow AC = 4$

ب)




$4^2 - 2^2 = 16 - 4 = 12 \rightarrow \sqrt{12} = 2\sqrt{3}$

$12 + (2\sqrt{3})^2 = 1 + 12 = 13 \rightarrow \sqrt{13}$

$\sin\alpha = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

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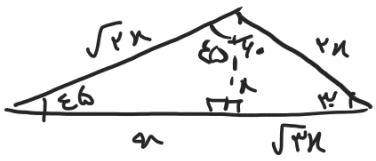


$(\sqrt{3}x)^2 - x^2 = 4x^2 - x^2 = 3x^2 \rightarrow \sqrt{3x^2} = \sqrt{3}x$

$x^2 + x^2 = 2x^2 \rightarrow \sqrt{2x^2} = \sqrt{2}x$

$$\frac{AB}{AC} = \frac{x + \sqrt{3}x}{\sqrt{2}x} = \frac{x(1+\sqrt{3})}{x\sqrt{2}} = \frac{1+\sqrt{3}}{\sqrt{2}}$$

$$\frac{AB}{AC} = \frac{1+\sqrt{3}}{2}$$

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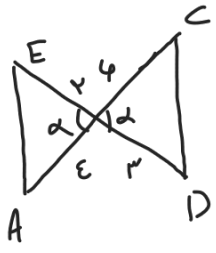
۵-




$S_{ABD} = \frac{1}{2} \times 2 \times 4 \times \sin\alpha = 4\sin\alpha$

$S_{BCD} = \frac{1}{2} \times 4 \times 3 \times \sin\alpha = 6\sin\alpha$

$$\frac{4\sin}{6\sin\alpha} = \frac{4}{6}$$

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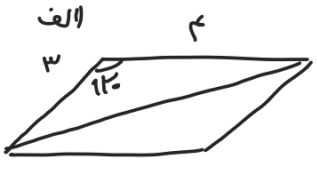
۶-

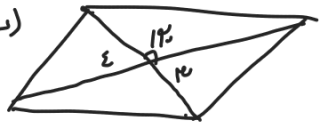
الف)




$\frac{1}{2} \times 3 \times 4 \times \sin 120 = \frac{12}{2}\sin 120 = \frac{6\sqrt{3}}{2}$

$3\sqrt{3} \times 2 = 6\sqrt{3}$

ب)




$\frac{1}{2} \times 3 \times 4 \times \sin 120 = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$

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۷-


$$\text{شیب} = \frac{\Delta y}{\Delta x} = \frac{4-2}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$$

$\tan\alpha = \frac{1}{2}$ $\quad 1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \rightarrow 1 + \frac{1}{4} = \frac{5}{4} \rightarrow \frac{1}{\cos^2\alpha}$

$\cos\alpha = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\quad \cos^2\alpha \rightarrow \frac{4}{5} \rightarrow \cos\alpha = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

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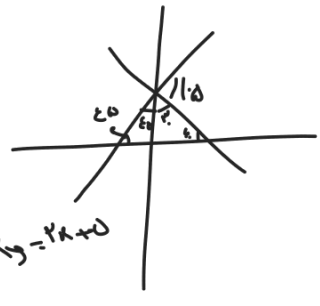
۸-




$2y = 2x + 5 \rightarrow y = x + 2.5$

$\tan 120 = \frac{\sin 120}{\cos 120} = \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}} = -\sqrt{3}$

$\tan\alpha = 1 \rightarrow \alpha = 45$

$y = mx + b \rightarrow m = -\sqrt{3}$

$mb = -\frac{5\sqrt{3}}{2} = -2.5\sqrt{3}$

$y = x + 2.5 = 7 \quad y = 6$

$y = -\sqrt{3}x + b \rightarrow 6 = -\sqrt{3} \times 0 + b \rightarrow b = 6$

$f(x) = \cos(\pi + x) + \sin\left(\frac{\pi}{2} - x\right) - \tan\left(\frac{3\pi}{2} + x\right)$

$\cos(\pi + x) = -\cos x$

$\sin\left(\frac{\pi}{2} - x\right) = \cos x \rightarrow -\cos x + \cos x + \cot x = \cot x$

$\tan\left(\frac{3\pi}{2} + x\right) = -\cot x$

$f(x) = \cot x$

$f\left(\frac{5\pi}{6}\right) = f(150) = \cot 150 = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

$\cot 150 = -\sqrt{3}$

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$$\frac{\sin\left(\frac{17\pi}{2} + x\right) + \cos\left(\frac{15\pi}{2} - x\right)}{\cos(3\pi + x) - \sin(4\pi - x)}$$

$\sin\left(\frac{17\pi}{2} + x\right) = \sin\frac{17\pi}{2}\cos x + \sin x \cos\frac{17\pi}{2}$ $\quad \cos\left(\frac{17\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

$\sin\left(\frac{17\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1 \rightarrow 1 \times \cos x + 0 \times \sin x = \cos x$

$\cos(3\pi + x) = \cos(\pi + x) = -\cos x$ / $\sin(4\pi - x) = \sin(2\pi - x) = -\sin x$

$\cos\left(\frac{15\pi}{2} - x\right) = \cos\frac{15\pi}{2}\cos x + \sin\frac{15\pi}{2}\sin x$ / $\cos\frac{15\pi}{2} = \cos\left(\frac{12\pi}{2} + \frac{3\pi}{2}\right) = \cos\left(4\pi + \frac{3\pi}{2}\right) = \cos\frac{3\pi}{2} = 0$

$\sin\frac{15\pi}{2} = \sin\left(\frac{12\pi}{2} + \frac{3\pi}{2}\right) = \sin\frac{3\pi}{2} = -1$

$$\frac{\cos x - \sin x}{-\cos x + \sin x} = \frac{\cos x - \sin x}{-\cos x + \sin x} = \frac{\cos - \sin}{-1(\cos - \sin)} = -1$$