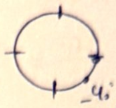
درس ریاضی، تکلیف شماره ۲۰، دهم تجربی.

۲۰

(۱)

الف) $(\sin 135 + \sin 300)(\cos 315 - \cos 150) = \left(\frac{\sqrt2}{2} - \frac{\sqrt3}{2}\right)\left(\frac{\sqrt2}{2} + \frac{\sqrt3}{2}\right) = \frac{(\sqrt2-\sqrt3)(\sqrt2+\sqrt3)}{4} = -\frac{1}{4}$

زاویه‌های مکمل $\sin 135 = \sin 45 = \frac{\sqrt2}{2}$

$\sin 300 = \sin(2\pi - 60) =$  $\to \sin x = -\sin(-x) \Rightarrow \sin_{-60} = -\sin 60 = -\frac{\sqrt3}{2}$

$\sin 300 = \sin(-60)$

$\cos 315 = \cos(-45) \Rightarrow \quad \cos x = \cos(-x) \Rightarrow$

$\cos 315 = \cos 45 = \frac{\sqrt2}{2}$ زاویه مکمل $\cos 150 = -\cos 30 = -\frac{\sqrt3}{2}$

ب) $\frac{1 + 3\tan^2 210}{4\cos^2 330 + 2\cos^2 225} = \frac{1 + 3\left(\frac{1}{3}\right)}{4\left(\frac{3}{4}\right) + 2\left(\frac{2}{4}\right)} = \frac{2}{4} = \frac{1}{2}$

$\tan 210 = \tan 30 = \frac{\sqrt3}{3}$

$\cos 330 = \cos(-30) = \cos(30) = \frac{\sqrt3}{2}$

$\cos 225 = -\cos 45 = -\frac{\sqrt2}{2}$

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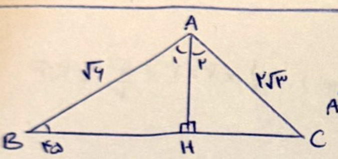
(۲) الف)



$HC = ?$

$\overset{\triangle}{ABH}: A_1 + \hat H + \hat B = 180 \to A_1 = 180 - 45 - 90 = 45$

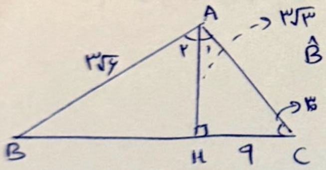
$45 = \hat A_1 = \hat B \Rightarrow AH = BH$

$AH = \sqrt6 \times \sin 45 = \sqrt6 \times \frac{\sqrt2}{2} = \frac{2\sqrt3}{2} = \sqrt3$

$\overset{\triangle}{ACH} = AH^2 + HC^2 = AC^2 \Rightarrow 3 + HC^2 = 12$

$HC^2 = 9 \to HC = \boxed{3}$

ب)



$\hat B = ?$

$\overset{\triangle}{ACH}: \hat A_1 + H + C = 180 \Rightarrow A_1 = 60$

$\tan 30 = \frac{AH}{CH} = \frac{\sqrt3}{3} \Rightarrow \frac{AH}{9} = \frac{\sqrt3}{3} \to AH = 3\sqrt3$

$\sin \hat B = \frac{AH}{AB} = \frac{3\sqrt3}{3\sqrt6} = \frac{1}{\sqrt2} = \frac{\sqrt2}{2} \Rightarrow \hat B = 45$

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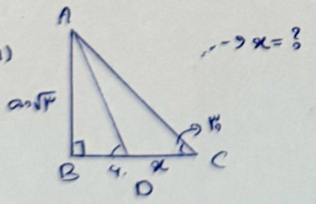
(۳) الف)



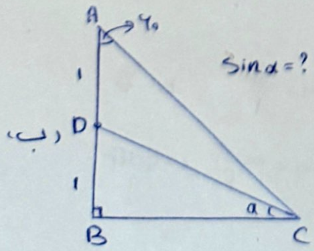
$x = ?$

$\tan 60 = \frac{50\sqrt3}{BD} = \sqrt3 \to BD = 50$

$\tan \hat C = \tan 30 = \frac{AB}{BC} = \frac{\sqrt3}{3} \to \frac{50\sqrt3}{BC} = \frac{\sqrt3}{3} \to BC = 150$

$DC = x = BC - BD = 150 - 50 = 100$

ب)



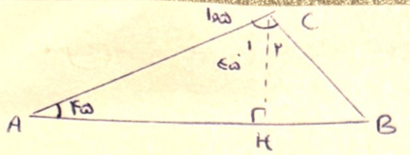
$\sin \alpha = ?$

$\tan 60 = \frac{BC}{AB} = \frac{BC}{2} = \frac{\sqrt3}{1} \to BC = 2\sqrt3$

$DC^2 = BD^2 + BC^2 = 12 + 1 \Rightarrow DC = \sqrt{13}$

$\sin \alpha = \frac{BD}{DC} = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

(4)



$C_1 = 180 - 45 - 90 = 45°$

$C_2 = 105 - 45 = 60°$

$AH = AC \times \frac{\sqrt{2}}{2}$ $\quad \hat{A} = C_1 \Rightarrow AH = CH = AC \times \frac{\sqrt{2}}{2}$

$C_2 = 60° \rightarrow \tan 60 = \frac{\sqrt{3}}{1} = \frac{BH}{CH} \rightarrow \frac{BH}{\frac{\sqrt{2}}{2}AC} = \sqrt{3} \rightarrow BH = \frac{\sqrt{6}}{2}AC$

$AB = BH + AH = \frac{\sqrt{6}}{2}AC + \frac{\sqrt{2}}{2}AC = \frac{\sqrt{6}+\sqrt{2}}{2}AC$ ---> نسبت $= \frac{\sqrt{6}+\sqrt{2}}{2}$

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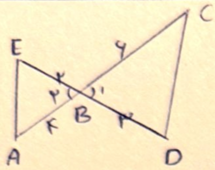
(5)



$\hat{B}_1 = \hat{B}_2$ (متقابل به رأس) $\quad (S_{A\hat{B}C} = \frac{1}{2} ab \sin \hat{C})$

$\frac{S_{ABE}}{S_{BCD}} = \frac{\cancel{\frac{1}{2}} \times \cancel{\sin \hat{B}_2} \times 2 \times 4}{\cancel{\frac{1}{2}} \times \cancel{\sin \hat{B}_1} \times 3 \times 6} = \frac{4}{9}$

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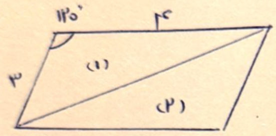
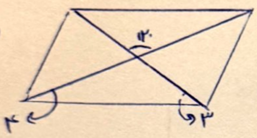
6) الف)



کل $S = S_1 + S_2$ $\quad (S_1 = S_2)$ $\quad \frac{\sqrt{3}}{2}$

$S_1 = \frac{1}{2} \times \sin_{120} \times 3 \times 4 = 6 \times \sin 120 = 3\sqrt{3}$ $\quad$ کل $S = 6\sqrt{3}$

ب)



$S_{\square} = \frac{1}{2} \times$ قطر (۱) $\times$ قطر (۲) $\times \sin \alpha$ (زاویه بین دو قطر) $= \frac{1}{2} \times 4 \times 3 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$

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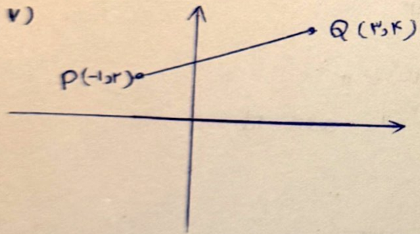
7)



$y = ax + b \rightarrow y = \frac{1}{2}x + b \xrightarrow{(3,4)} 4 = \frac{3}{2} + b \rightarrow b = \frac{5}{2}$

$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{3+1} = \frac{2}{4} = \frac{1}{2}$

$y = \frac{1}{2}x + \frac{5}{2} \rightsquigarrow a = \tan\theta = \frac{1}{2}$

$\cos\theta = ?$ $\quad 1 + \tan^2\theta = \frac{1}{\cos^2\theta} \Rightarrow \underbrace{1 + \frac{1}{4}}_{\frac{5}{4}} = \frac{1}{\cos^2\theta} \rightarrow \cos\theta = \frac{\pm 2}{\sqrt{5}}$

$\Rightarrow \cos\theta = \frac{+2\sqrt{5}}{5}$

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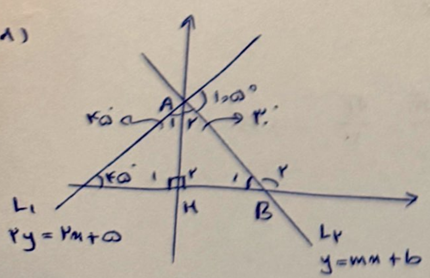
8)



$L_1: \tan\theta = \frac{2}{2} = 1 \rightarrow \theta = 45°$

$\Rightarrow A_1 = 45°$ $\quad \hat{A}_1 + \hat{A}_2 = 180 - 105 = 75°$

$\Rightarrow A_2 = 30°$

$\hat{B}_1 = 180 - 90 - 30 = 60°$

$\hat{B}_2 = 180 - 60 = 120°$

$\tan_{(L_2)} = \tan 120° = -\sqrt{3}$ $\quad \Rightarrow m = -\sqrt{3}$ $\quad b = \frac{5}{2}$

$\hat{A} = (0, y) \rightarrow 2y = 2x + 5 \xrightarrow{x=0} 2y = 5$

$y = \frac{5}{2}$

$mb = \frac{-5\sqrt{3}}{2} = -2/5\sqrt{3}$

9) $f(x) = \cos(\pi + x) + \sin(\frac{\pi}{2} - x) - \tan(\frac{3\pi}{2} + x) = \cancel{-\cos x} + \cancel{\cos x} + \cot x \Rightarrow f(x) = \cot(x)$

$$f(\frac{5\pi}{6}) = \cot(\frac{5\pi}{6}) = \cot(150^\circ) = -\sqrt{3}$$

$\frac{16\pi}{2} + \frac{\pi}{2} = 8\pi + \frac{\pi}{2} \qquad \frac{14\pi}{2} + \frac{\pi}{2} = 7\pi + \frac{\pi}{2}$

(10) $$\frac{\sin(\frac{17\pi}{2} + x) + \cos(\frac{15\pi}{2} - x)}{\cos(3\pi + x) - \sin(6\pi - x)} = \frac{+\cos x - \sin x}{-\cos x + \sin x} = -1$$