کیمیا نصراصفهانی - دهم دختر C - تکلیف ۲۱

① $1! = 1 \quad 2! = 2$

$3! = 6 \quad 4! = 24$ $\Big\} \to (7)(26) \to 182 \to \boxed{2}$

$5! = 0 \quad 6! = 0 \quad \vdots$ از اینجا به بعد صفر

② الف) برای تبدیل به مربع (۴)

$$\sqrt[4]{(1+\sqrt7)^2} \to \sqrt[4]{1+7+2\sqrt7} \leadsto \sqrt[4]{\frac{8+2\sqrt7}{4+\sqrt7}} = \sqrt[4]{\frac{2(\cancel{4+\sqrt7})}{\cancel{4+\sqrt7}}} = \sqrt[4]{2}$$

ب)
$$\frac{(\sqrt2+\sqrt5)(\sqrt{10}-2)}{\cancel{10-4}\ 6} \leadsto \frac{\cancel{\sqrt{20}}^{\,2\sqrt5} - 2\sqrt2 + \sqrt{50}^{\,5\sqrt2} - \cancel{2\sqrt5}}{} = \frac{3\sqrt2}{6} = \frac{\sqrt2}{2}$$

$$\frac{\sqrt2}{2}\left(\sqrt{3-\sqrt5} - \sqrt{3+\sqrt5}\right) \quad \frac{\sqrt{6-2\sqrt5} - \sqrt{6+2\sqrt5}}{2} \xrightarrow{\text{مربع}} \sqrt{(\sqrt5-\sqrt1)^2}$$

$$\Rightarrow \frac{(\sqrt5-\sqrt1)-(\sqrt5+\sqrt1)}{2} = \frac{\sqrt5-1-\sqrt5-1}{2} = \frac{-2}{2} = \boxed{-1}$$

③ الف)
$$\frac{(2\sqrt2+3\sqrt3)(5+\sqrt6)}{\cancel{25-6}\ 19} \to \frac{10\sqrt2 + \cancel{2\sqrt{12}}^{\,4\sqrt3} + 15\sqrt3 + \cancel{3\sqrt{18}}^{\,9\sqrt2}}{19}$$

$$\frac{\cancel{19}\sqrt2 + \cancel{19}\sqrt3}{\cancel{19}} = \sqrt2+\sqrt3 - \boxed{\frac{2}{\sqrt[4]{3^2}-1}} \xrightarrow{\text{گویا کردن}} \times\frac{\sqrt3+1}{\sqrt3+1} \cdot \frac{2\sqrt3+2}{\cancel{3-1}\ 2} = \sqrt3+1$$

$$\sqrt2+\cancel{\sqrt3}-\cancel{\sqrt3}-1 = \sqrt2-1$$

ب)
$$\frac{(3\sqrt3-1)}{4+\sqrt3}\times\frac{(4-\sqrt3)}{4-\sqrt3} = \frac{12\sqrt3-9-4+\sqrt3}{16-3} = \frac{13\sqrt3-13}{13} = \sqrt3-1$$

$$(2-\sqrt3)^{-1} \to \frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3} = \frac{2+\sqrt3}{\cancel{4-3}\ 1} \Rightarrow \sqrt3-1+2+\sqrt3 = 2\sqrt3+1$$

۴) الف) $\dfrac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}} - 2$

$(\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1})^2 = A^2 \to 1+\sqrt3+\sqrt3-1+2\left(\sqrt{\underset{2}{\cancel{3-1}}}\right) = \sqrt{2\sqrt3+2\sqrt2} \to \boxed{A}$

$\sqrt{\dfrac{2\sqrt3+2\sqrt2}{\sqrt3-\sqrt2}\times\dfrac{\sqrt3+\sqrt2}{\sqrt3+\sqrt2}} \to \sqrt{\dfrac{6+2\sqrt6+2\sqrt6+4}{\underset{1}{\cancel{3-2}}}} = \sqrt{10+4\sqrt6}$

$\sqrt{10+4\sqrt6} \xrightarrow{\text{بام}} \sqrt{(\sqrt6+\sqrt4)^2} = \sqrt6+\overset{2}{\cancel{\sqrt4}}-2 = \boxed{\sqrt6}$

ب) $\underbrace{2^{\frac{8}{12}}}\times 3^1\times\underbrace{2^{\frac14}}\times\underbrace{2^1}\times\underbrace{2^{\frac{1}{12}}} = 2^{\frac{8}{12}+\frac14+1+\frac{1}{12}}\times 3 = 2^2\times 3 = \boxed{12}$

۵) $\dfrac{3^x+3^x\times3+3^x\times3^2+3^x\times3^3+3^x\times3^4+3^x\times3^5}{2^x\times2^{-2}+2^x\times2^{-1}+2^x+2^x\times2+2^x\times2^2+2^x\times2^3}$

$\dfrac{3^x(\cancel{1+3+9+27+81+243})^{\boxed{364}}}{2^{x-2}(\underbrace{\cancel{1+2+4+\dots+2^5}}_{63})} = 52 \to \dfrac{3^x\times364}{2^{x-2}\times63} = 52$

$3^x\times364 = 52\times2^x\times\dfrac14\times63 \to \dfrac{3^x}{2^x} = \boxed{2/25} \to \left(\dfrac32\right)^x = \dfrac94 \to x=2$

۶) $((a-b)^2)^2((a+b)^2)^2 \to \overbrace{(a-b)^4(a+b)^4}^{\text{مزدوج}} = (a^2-b^2)^4$

$\left(\sqrt{\sqrt6-2}-\sqrt{\sqrt6+2}\right)^4 \xrightarrow{(1)^2} \left(\sqrt6-2+\sqrt6+2-2\sqrt{\overset{2\sqrt2}{\cancel{6-4}}}\right)^2$

$(2\sqrt6-2\sqrt2)^2 = \underset{32}{\cancel{24+8}}-\underset{16\sqrt3}{\cancel{8\sqrt{12}}} \to 16(2-\sqrt3) \to \boxed{t=16}$

۷) $(a+\frac{۱}{a}+\sqrt{۲})^{۲}(a+\frac{۱}{a}-\sqrt{۲})^{۲} \to ((a+\frac{۱}{a})^{۲}-۲)^{۲}$

$(a^{۲}+\frac{۱}{a^{۲}}+\cancel{۲-۲})^{۲} \to (a^{۲}+\frac{۱}{a^{۲}})^{۲} = a^{۴}+\frac{۱}{a^{۴}}+۲$

$۷-۴\sqrt{۳}+\frac{۱}{۷-۴\sqrt{۳}}+۲ \to ۷-\cancel{۴\sqrt{۳}}+۷+\cancel{۴\sqrt{۳}}+۲ = ۱۶ = ۲^{t} \to \boxed{t=۴}$

۸) $۲^{\frac{۲}{۵}}\times ۲^{\frac{۴}{۱۵}}\times ۲^{\frac{۴}{۳}} = A \to A = ۲^{\frac{۲}{۵}+\frac{۴}{۱۵}+\frac{۴}{۳}} = ۲^{۲}$

$(۸)^{-\frac{۱}{۳}} \to (۲^{۳})^{-\frac{۱}{۳}} = ۲^{-۱} = \frac{۱}{۲}$

۹) $\sqrt[۷]{a} = ۲۷\times a^{\frac{۱۵}{۷}} \xrightarrow{(\,)^{۷}} a = ۲۷^{۷}\times a^{۱۵} \to a^{۱۴} = ۲۷^{-۷} \to a^{۱۴} = ۳^{-۲۱}$

$\frac{a}{a^{۱۵}} = a^{-۱۴}$

$\xrightarrow{(\,)^{\frac{۱}{۱۴}}} a = ۳^{\frac{-۲۱}{۱۴}=\frac{-۳}{۲}}$

$۳^{-\frac{۳}{۲}} = \frac{۱}{۳\sqrt{۳}} \to (۳\sqrt{۳}-۳) \to \frac{۳(\cancel{\sqrt{۳}}-۱)}{\cancel{\sqrt{۳}}-۳} =$ ۳ برابر

۱۰) $\sqrt{x+a} = ۲+\sqrt{x-۴} \xrightarrow{(\,)^{۲}} \cancel{x}+a = \cancel{۴}+\cancel{x}-\cancel{۴}+۴\sqrt{x-۴} \Rightarrow \sqrt{x-۴} = \frac{a}{۴}$

جایگذاری

$۲+\sqrt{x-۴}+\sqrt{x-۴}-۲ \to ۲\sqrt{x-۴} = ۲\times\frac{a}{۴} = \boxed{\frac{a}{۲}}$