مهرانا فراغتی - دهم دختر C - تکلیف شماره ی ۲۱

۱۹٫۵

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$$(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+24!)=(1+6+0+\dots+0)(2+4+0+\dots+0)=$$
$$7\times6=42 \rightsquigarrow \boxed{2}$$

(۲) ✓

۲

$$\sqrt[4]{(4+\sqrt7)^{-1}}\ \sqrt{1+\sqrt7}=\sqrt[4]{(4+\sqrt7)^{-1}}\times\sqrt[4]{(1+\sqrt7)^2}=\sqrt[4]{\frac{1}{(4+\sqrt7)}}\times\sqrt[4]{1+7+2\sqrt7}$$
$$=\sqrt[4]{\frac{1}{4+\sqrt7}}\times\sqrt[4]{8+2\sqrt7}=\sqrt[4]{\frac{2(4+\sqrt7)}{(4+\sqrt7)}}=\boxed{\sqrt[4]{2}}$$

✓ ۱٫۵

$$\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)=$$

$$b^2=(3-\sqrt5)+(3+\sqrt5)-2\sqrt{9-5}$$
$$b^2=6-4 \to b^2=2 \to b=\pm\sqrt2$$

$$\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)^2=3-\sqrt5+3+\sqrt5-2\sqrt{9-5}=6-2\sqrt4=6-4=2 \rightsquigarrow -\sqrt2$$

$$\frac{\sqrt2+\sqrt5}{\sqrt2(\sqrt2+\sqrt5)}=\frac{1}{\sqrt2}\times\frac{\sqrt2}{\sqrt2}=\frac{\sqrt2}{2} \rightsquigarrow \frac{\sqrt2}{2}\times-\sqrt2=\frac{-2}{2}=\boxed{-1}$$

۳

$$\frac{\sqrt8+\sqrt{27}}{5-\sqrt6}-2\left(\sqrt[4]{9}-1\right)^{-1}=$$

$$5-\sqrt6 \to (\sqrt2)^3+(\sqrt3)^3=(\sqrt2+\sqrt3)(2+3+\sqrt6)$$

$$(\sqrt2)^3+(\sqrt3)^3=(\sqrt2+\sqrt3)(5+\sqrt6) \rightsquigarrow 5+\sqrt6=\frac{(\sqrt2)^3+(\sqrt3)^3}{\sqrt2+\sqrt3}$$

$$\frac{\cancel{\sqrt8+\sqrt{27}}}{\frac{1}{\frac{\cancel{\sqrt8+\sqrt{27}}}{\sqrt2+\sqrt3}}}=\sqrt2+\sqrt3 \quad \left\{ \begin{array}{l} 2(\sqrt[4]{9}-1)^{-1}=2(\sqrt3-1)^{-1}=\frac{2}{\sqrt3-1}\times\frac{(\sqrt3+1)}{(\sqrt3+1)}= \\ \frac{2(\sqrt3+1)}{3-1}=\frac{2(\sqrt3+1)}{2}=\sqrt3+1 \end{array}\right.$$

✓

$$\sqrt2+\cancel{\sqrt3}-\cancel{\sqrt3}-1=\boxed{\sqrt2-1}$$

(۲)

$$\frac{\sqrt{27}-1}{4+\sqrt3}+(2-\sqrt3)^{-1}$$

$$4+\sqrt3 \to \sqrt3^{\,3}-1=(\sqrt3-1)(3+1+\sqrt3) \rightsquigarrow \sqrt3^{\,3}-1=(\sqrt3-1)(4+\sqrt3)$$

$$4+\sqrt3=\frac{\sqrt{27}-1}{\sqrt3-1} \to \frac{\cancel{\sqrt{27}-1}}{\frac{1}{\frac{\cancel{\sqrt{27}-1}}{\sqrt3-1}}}=\sqrt3-1 \quad \left\{ \frac{1}{2-\sqrt3}\times\frac{(2+\sqrt3)}{(2+\sqrt3)}=\frac{2+\sqrt3}{4-3}=2+\sqrt3 \right.$$

✓

s.a.m

$$\sqrt3-1+2+\sqrt3=\boxed{2\sqrt3+1}$$

④

$$\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}-2$$

↓ توان ۲

$$\left(\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}\right)^2=\frac{1+\sqrt{3}+\sqrt{3}-\cancel{1}+2\sqrt{3-1}}{\sqrt{3}-\sqrt{2}}=\frac{2\sqrt{3}+2\sqrt{2}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=$$

$$\frac{2(\sqrt{3}+\sqrt{2})^2}{3-2}=\frac{2(3+2+2\sqrt{6})}{1}=2(5+2\sqrt{6})=10+4\sqrt{6}=(\sqrt{6}+2)^2$$

$$\xrightarrow{\text{جذر}}\sqrt{(\sqrt{6}+2)^2}=\sqrt{6}+2\rightsquigarrow\sqrt{6}+2-2=\boxed{\sqrt{6}}$$ ✓

②

$$\sqrt[4]{\sqrt[3]{2^8}}\times\sqrt[4]{162}\times\sqrt{4\sqrt[6]{2}}=\sqrt[12]{2^8}\times\sqrt[4]{2\times81}\times\sqrt{4}\times\sqrt[12]{2}=$$

$$\sqrt[12]{2^9}\times2\times3\sqrt[4]{2}=\sqrt[4]{2^3}\times2\times3\sqrt[4]{2}=\sqrt[4]{2^4}\times6=2\times6=\boxed{12}$$ ✓

⑤

$$\frac{3^x+3^{x+1}+3^{x+2}+3^{x+3}+3^{x+4}+3^{x+5}}{2^{x-2}+2^{x-1}+2^x+2^{x+1}+2^{x+2}+2^{x+3}}=52$$

$$3^x(1+3+9+27+81+243)=3^x\times364$$

$$2^x\left(\frac{1}{4}+\frac{1}{2}+1+2+4+8\right)=2^x\left(15\frac{3}{4}\right)=2^x\times\frac{63}{4}$$ ② ✓

$$\frac{3^x\times364}{2^x\times\frac{63}{4}}=52\rightsquigarrow3^x\times\overset{28}{\cancel{364}}=2^x\times\frac{63}{\cancel{4}}\times\overset{4}{\cancel{52}}\rightsquigarrow3^x\times\overset{4}{\cancel{28}}=2^x\times\overset{9}{\cancel{63}}$$

$$3^x\times4=2^x\times9\rightsquigarrow3^x\times2^2=2^x\times3^2\rightsquigarrow\boxed{x=2}$$

⑥

$$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2=[(a-b)^2]^2[(a+b)^2]^2=(a-b)^4(a+b)^4$$

$$((a^2-b^2)^2)^2\to a^2-b^2=\sqrt{\sqrt{6}-2}-\sqrt{\sqrt{6}+2}\rightsquigarrow\left(\sqrt{\sqrt{6}-2}-\sqrt{\sqrt{6}+2}\right)^2=$$

$$\sqrt{6}-\cancel{2}+\sqrt{6}+\cancel{2}-2\sqrt{6-4}=2\sqrt{6}-2\sqrt{2}\rightsquigarrow[2(\sqrt{6}-\sqrt{2})]^2=4(6+2-2\sqrt{12})$$

$$4(8-4\sqrt{3})=4\times4(2-\sqrt{3})=16(2-\sqrt{3})=t(2-\sqrt{3})$$

s.a.m

$$\boxed{t=16}$$ ② ✓

$$\left(a+\frac{1}{a}+\sqrt{۲}\right)^{۲}\left(a+\frac{1}{a}-\sqrt{۲}\right)^{۲}=۲^{t}$$ ۷

$$\left(\left[\left(a+\frac{1}{a}\right)+\sqrt{۲}\right]\left[\left(a+\frac{1}{a}\right)-\sqrt{۲}\right]\right)^{۲}=\left(a^{۲}+\frac{1}{a^{۲}}+\cancel{۲}-\cancel{۲}\right)^{۲}=\left(a^{۲}+\frac{1}{a^{۲}}\right)^{۲}=$$

$$a^{۴}+\frac{1}{a^{۴}}+۲=۷-۴\sqrt{۳}+\frac{1}{۷-۴\sqrt{۳}}+۲ \rightarrow ۷-\cancel{۴\sqrt{۳}}+۷+\cancel{۴\sqrt{۳}}+۲=۱۶$$

$$\frac{1}{۷-۴\sqrt{۳}}\times\frac{(۷+۴\sqrt{۳})}{(۷+۴\sqrt{۳})}=\frac{۷+۴\sqrt{۳}}{۴۹-۴۸}=۷+۴\sqrt{۳}$$

$$۱۶=۲^{t}\rightarrow ۲^{۴}=۲^{t}$$

$$\boxed{t=۴}$$

۲

$$A=\sqrt[۵]{۴\sqrt[۳]{۱۶}}\left(\frac{1}{۲}\right)^{-\frac{۴}{۳}}$$ ۸

$$\sqrt[۵]{۴}\times\sqrt[۱۵]{۱۶}\times ۲^{\frac{۴}{۳}}=\sqrt[۵]{۲^{۲}}\times\sqrt[۱۵]{۲^{۴}}\times\sqrt[۳]{۲^{۴}}=$$

$$\sqrt[۱۵]{۲^{۶}}\times\sqrt[۱۵]{۲^{۴}}\times\sqrt[۱۵]{۲^{۲۰}}=\sqrt[۱۵]{۲^{۳۰}}=۲^{\frac{۳۰}{۱۵}}=۲^{۲}=۴\rightarrow A=۴$$

$$(۲A)^{-\frac{1}{۳}}=(۲\times۴)^{-\frac{1}{۳}}=(۲^{۳})^{-\frac{1}{۳}}=۲^{۳\times(-\frac{1}{۳})}=۲^{-۱}=\boxed{\frac{1}{۲}}$$

۲

$$\sqrt[۷]{a}=۲۷\times a^{\frac{۱۵}{۷}}\rightarrow a^{\frac{1}{۷}}=۲۷\times a^{\frac{۱۵}{۷}}$$ ۹

$$۱=۲۷\times\frac{a^{\frac{۱۵}{۷}}}{a^{\frac{1}{۷}}}\rightarrow ۱=۲۷\times a^{\frac{۱۵}{۷}-\frac{1}{۷}}=۲۷\times a^{\frac{۱۴}{۷}}$$

$$۱=۲۷\times a^{۲}\rightarrow a^{۲}=\frac{1}{۲۷}\rightarrow a=\pm\frac{1}{۳\sqrt{۳}}\rightarrow a=+\frac{1}{۳\sqrt{۳}}$$

$$\left(\frac{1}{a}-۳\right)=۳\sqrt{۳}-۳=۳(\sqrt{۳}-۱)$$

$$\frac{۳(\sqrt{۳}-۱)}{(\sqrt{۳}+۱)}\times\frac{(\sqrt{۳}-۱)}{(\sqrt{۳}-۱)}=\frac{۳(۳+۱-۲\sqrt{۳})}{۳-۱}=\frac{۳(۴-۲\sqrt{۳})}{۲}=$$

$$\frac{۳\times\cancel{۲}(۲-\sqrt{۳})}{\cancel{۲}}=۳(۲-\sqrt{۳})=\boxed{۶-۳\sqrt{۳}}$$

۲

$\sqrt{x+a} - \sqrt{x-۴} = ۲$ ۱۰

$\sqrt{x+a} + \sqrt{x-۴} - ۲ = ?$

$\underbrace{\cancel{\left(\sqrt{x+a} - \sqrt{x-۴}\right)}}_{}^{۲}\left(\sqrt{x+a} + \sqrt{x-۴}\right) = x+a-(x-۴)$

$\cancel{x} + a - \cancel{x} + ۴ = a + ۴$

$\sqrt{x+a} + \sqrt{x-۴} = \frac{a+۴}{۲} = \frac{a}{۲} + ۲$ ۴ ✓

$\frac{a}{۲} + ۲ - ۲ = \boxed{\frac{a}{۲}}$