مهرانا فراغتی - دهم دختر C - تکلیف شماره‌ی ۲۱

①

$$(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+24!)=(1+6+0+\dots+0)(2+4+0+\dots+0)=$$
$$7\times 6=42 \rightsquigarrow \boxed{2}$$

②

$$\sqrt[4]{(4+\sqrt{7})^{-1}}\ \sqrt{1+\sqrt{7}}=\sqrt[4]{(4+\sqrt{7})^{-1}}\times\sqrt[4]{(1+\sqrt{7})^{2}}=\sqrt[4]{\frac{1}{(4+\sqrt{7})}}\times\sqrt[4]{1+7+2\sqrt{7}}$$
$$=\sqrt[4]{\frac{1}{4+\sqrt{7}}}\times\sqrt[4]{8+2\sqrt{7}}=\sqrt[4]{\frac{2(4+\sqrt{7})}{(4+\sqrt{7})}}=\boxed{\sqrt[4]{2}}$$

$$\left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}}\right)=$$

$$\left(\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}}\right)^{2}=3-\sqrt{5}+3+\sqrt{5}-2\sqrt{9-5}=6-2\sqrt{4}=6-4=2\rightsquigarrow\sqrt{2}$$

$$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}\,(\sqrt{2}+\sqrt{5})}=\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}\rightsquigarrow\frac{\sqrt{2}}{2}\times\sqrt{2}=\frac{2}{2}=\boxed{1}$$

③

$$\frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{6}}-2\left(\sqrt[4]{9}-1\right)^{-1}=$$

$\downarrow$

$$5-\sqrt{6}\rightarrow(\sqrt{2})^{3}+(\sqrt{3})^{3}=(\sqrt{2}+\sqrt{3})(2+3+\sqrt{6})$$

$$(\sqrt{2})^{3}+(\sqrt{3})^{3}=(\sqrt{2}+\sqrt{3})(5+\sqrt{6})\rightsquigarrow 5+\sqrt{6}=\frac{(\sqrt{2})^{3}+(\sqrt{3})^{3}}{\sqrt{2}+\sqrt{3}}$$

$$\frac{\frac{\cancel{\sqrt{8}+\sqrt{27}}}{1}}{\frac{\cancel{\sqrt{8}+\sqrt{27}}}{\sqrt{2}+\sqrt{3}}}=\sqrt{2}+\sqrt{3}$$

$$\left\{\begin{aligned}&2(\sqrt[4]{9}-1)^{-1}=2(\sqrt{3}-1)^{-1}=\frac{2}{\sqrt{3}-1}\times\frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}=\\&\frac{2(\sqrt{3}+1)}{3-1}=\frac{2(\sqrt{3}+1)}{2}=\sqrt{3}+1\end{aligned}\right.$$

$$\sqrt{2}+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}-1=\boxed{\sqrt{2}-1}$$

$$\frac{\sqrt{27}-1}{4+\sqrt{3}}+(2-\sqrt{3})^{-1}$$

$$4+\sqrt{3}\rightarrow\sqrt{3}^{3}-1=(\sqrt{3}-1)(3+1+\sqrt{3})\rightsquigarrow\sqrt{3}^{3}-1=(\sqrt{3}-1)(4+\sqrt{3})$$

$$4+\sqrt{3}=\frac{\sqrt{27}-1}{\sqrt{3}-1}\rightarrow\frac{\frac{\cancel{\sqrt{27}-1}}{1}}{\frac{\cancel{\sqrt{27}-1}}{\sqrt{3}-1}}=\sqrt{3}-1\quad\left\{\frac{1}{2-\sqrt{3}}\times\frac{(2+\sqrt{3})}{(2+\sqrt{3})}=\frac{2+\sqrt{3}}{4-3}=2+\sqrt{3}\right.$$

s.a.m

$$\sqrt{3}-1+2+\sqrt{3}=\boxed{2\sqrt{3}+1}$$

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$$\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}-2$$

$\downarrow$ توان دو

$$\left(\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}\right)^2=\frac{1+\sqrt{3}+\sqrt{3}-\cancel{1}+2\sqrt{3-1}}{\sqrt{3}-\sqrt{2}}=\frac{2\sqrt{3}+2\sqrt{2}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=$$

$$\frac{2(\sqrt{3}+\sqrt{2})^2}{3-2}=\frac{2(3+2+2\sqrt{6})}{1}=2(5+2\sqrt{6})=10+4\sqrt{6}=(\sqrt{6}+2)^2$$

$$\xrightarrow{\text{جذر}}\sqrt{(\sqrt{6}+2)^2}=\sqrt{6}+2\rightsquigarrow\sqrt{6}+2-2=\boxed{\sqrt{6}}$$

$$\sqrt[4]{\sqrt[3]{2^8}}\times\sqrt[4]{162}\times\sqrt{4\sqrt[6]{2}}=\sqrt[12]{2^8}\times\sqrt[4]{2\times81}\times\sqrt{4}\times\sqrt[12]{2}=$$

$$\sqrt[12]{2^9}\times2\times3\sqrt[4]{2}=\sqrt[4]{2^3}\times2\times3\sqrt[4]{2}=\sqrt[4]{2^4}\times6=2\times6=\boxed{12}$$

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$$\frac{3^x+3^{x+1}+3^{x+2}+3^{x+3}+3^{x+4}+3^{x+5}}{2^{x-2}+2^{x-1}+2^x+2^{x+1}+2^{x+2}+2^{x+3}}=52$$

$$3^x(1+3+9+27+81+243)=3^x\times364$$

$$2^x\left(\frac{1}{4}+\frac{1}{2}+1+2+4+8\right)=2^x\left(15\frac{3}{4}\right)=2^x\times\frac{63}{4}$$

$$\frac{3^x\times364}{2^x\times\frac{63}{4}}=52\rightsquigarrow3^x\times\overset{28}{\cancel{364}}=2^x\times\frac{63}{\cancel{4}}\times\overset{4}{\cancel{52}}\rightsquigarrow3^x\times\overset{4}{\cancel{28}}=2^x\times\overset{9}{\cancel{63}}$$

$$3^x\times4=2^x\times9\rightsquigarrow3^x\times2^2=2^x\times3^2\rightsquigarrow\boxed{x=2}$$

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$$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2=[(a-b)^2]^2[(a+b)^2]^2=(a-b)^4(a+b)^4$$

$$((a^2-b^2)^2)^2\to a^2-b^2=\sqrt{\sqrt{6}-2}-\sqrt{\sqrt{6}+2}\rightsquigarrow\left(\sqrt{\sqrt{6}-2}-\sqrt{\sqrt{6}+2}\right)^2=$$

$$\sqrt{6}-\cancel{2}+\sqrt{6}+\cancel{2}-2\sqrt{6-4}=2\sqrt{6}-2\sqrt{2}\rightsquigarrow[2(\sqrt{6}-\sqrt{2})]^2=4(6+2-2\sqrt{12})$$

$$4(8-4\sqrt{3})=4\times4(2-\sqrt{3})=16(2-\sqrt{3})=t(2-\sqrt{3})$$

$$\boxed{t=16}$$

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$$(a+\frac{۱}{a}+\sqrt{۲})^{۲}(a+\frac{۱}{a}-\sqrt{۲})^{۲}=۲^{t}$$

$$\left(\left[(a+\frac{۱}{a})+\sqrt{۲}\right]\left[(a+\frac{۱}{a})-\sqrt{۲}\right]\right)^{۲}=(a^{۲}+\frac{۱}{a^{۲}}+\cancel{۲}-۲)^{۲}=(a^{۲}+\frac{۱}{a^{۲}})^{۲}=$$

$$a^{۴}+\frac{۱}{a^{۴}}+۲=۷-۴\sqrt{۳}+\frac{۱}{۷-۴\sqrt{۳}}+۲ \rightarrow ۷-\cancel{۴\sqrt{۳}}+۷+\cancel{۴\sqrt{۳}}+۲=۱۶$$

$$\frac{۱}{۷-۴\sqrt{۳}}\times\frac{(۷+۴\sqrt{۳})}{(۷+۴\sqrt{۳})}=\frac{۷+۴\sqrt{۳}}{۴۹-۴۸}=۷+۴\sqrt{۳}$$

$$۱۶=۲^{t}\rightarrow ۲^{۴}=۲^{t}$$

$$\boxed{t=۴}$$

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$$A=\sqrt[۵]{۴\sqrt[۳]{۱۶}}\;(\frac{۱}{۲})^{-\frac{۴}{۳}}$$

$$\sqrt[۵]{۴}\times\sqrt[۱۵]{۱۶}\times ۲^{\frac{۴}{۳}}=\sqrt[۵]{۲^{۲}}\times\sqrt[۱۵]{۲^{۴}}\times\sqrt[۳]{۲^{۴}}=$$

$$\sqrt[۱۵]{۲^{۶}}\times\sqrt[۱۵]{۲^{۴}}\times\sqrt[۱۵]{۲^{۲۰}}=\sqrt[۱۵]{۲^{۳۰}}=۲^{\frac{۳۰}{۱۵}}=۲^{۲}=۴\rightarrow A=۴$$

$$(۲A)^{-\frac{۱}{۳}}=(۲\times۴)^{-\frac{۱}{۳}}=(۲^{۳})^{-\frac{۱}{۳}}=۲^{۳\times(-\frac{۱}{۳})}=۲^{-۱}=\boxed{\frac{۱}{۲}}$$

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$$\sqrt[۷]{a}=۲۷\times a^{\frac{۱۵}{۷}}\rightarrow a^{\frac{۱}{۷}}=۲۷\times a^{\frac{۱۵}{۷}}$$

$$۱=۲۷\times\frac{a^{\frac{۱۵}{۷}}}{a^{\frac{۱}{۷}}}\rightarrow ۱=۲۷\times a^{\frac{۱۵}{۷}-\frac{۱}{۷}}=۲۷\times a^{\frac{۱۴}{۷}}$$

$$۱=۲۷\times a^{۲}\rightarrow a^{۲}=\frac{۱}{۲۷}\rightarrow a=\pm\frac{۱}{۳\sqrt{۳}}\rightarrow a=+\frac{۱}{۳\sqrt{۳}}$$

$$(\frac{۱}{a}-۳)=۳\sqrt{۳}-۳=۳(\sqrt{۳}-۱)$$

$$\frac{۳(\sqrt{۳}-۱)}{(\sqrt{۳}+۱)}\times\frac{(\sqrt{۳}-۱)}{(\sqrt{۳}-۱)}=\frac{۳(۳+۱-۲\sqrt{۳})}{۳-۱}=\frac{۳(۴-۲\sqrt{۳})}{۲}=$$

$$\frac{۳\times۲(۲-\sqrt{۳})}{\cancel{۲}}=۳(۲-\sqrt{۳})=\boxed{۶-۳\sqrt{۳}}$$

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$$\sqrt{x+a}-\sqrt{x-4}=2$$ 15

$$\sqrt{x+a}+\sqrt{x-4}-2=?$$

$$\cancel{\left(\sqrt{x+a}-\sqrt{x-4}\right)}^{2}\left(\sqrt{x+a}+\sqrt{x-4}\right)=x+a-(x-4)$$

$$\cancel{x}+a-\cancel{x}+4=a+4$$

$$\sqrt{x+a}+\sqrt{x-4}=\frac{a+4}{2}=\frac{a}{2}+2$$

$$\frac{a}{2}+2-2=\boxed{\frac{a}{2}}$$