

$$(1! + 3! + 5! + \dots + 25!) (2! + 4! + 6! + \dots + 24!)$$

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 $1 \quad 4 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2 \quad 2$

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الف) $\sqrt[3]{(4+\sqrt{7})^{-1}} \sqrt{1+\sqrt{7}} = \sqrt[3]{\frac{4-\sqrt{7}}{9} (1+\sqrt{7})^2} = \sqrt[3]{\frac{4-\sqrt{7}}{9} (1+2\sqrt{7})} = \sqrt[3]{\frac{4-1\sqrt{7}+2\sqrt{7}-14}{9}}$

$\frac{1}{4+\sqrt{7}} = \frac{4-\sqrt{7}}{9}$

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$= \sqrt[3]{2}$

ب) $\frac{(4+\sqrt{5})}{\sqrt{5}+2} (\sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}}) = \frac{\sqrt{5}}{2} \times \sqrt{2} = 1$

$\rightarrow \frac{(\sqrt{2}+\sqrt{5})(\sqrt{5}-2)}{(\sqrt{5}+2)} = \frac{2\sqrt{5}-2\sqrt{2}+5\sqrt{2}-4\sqrt{5}}{(\sqrt{5}+2)} = \frac{\sqrt{5}}{2}$

$A = \sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}}$
 $A^2 = 3-\sqrt{5} + 3+\sqrt{5} - 2\sqrt{9-5} = 6-2\sqrt{4} = 2$
 $A = \sqrt{2}$

الف) $\frac{\sqrt{1+\sqrt{2}}}{5-\sqrt{5}} - 2(\sqrt{9}-1)^{-1} \rightarrow \frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{2} \Rightarrow \sqrt{2+\sqrt{3}} - \sqrt{3-1} = \sqrt{2-1} = 1$

$\rightarrow \frac{(\sqrt{2}+1)(\sqrt{5}+5)}{(5-\sqrt{5})(\sqrt{5}+5)} = \frac{5\sqrt{2}+4\sqrt{3}+\sqrt{3}+9\sqrt{2}}{19} = \sqrt{2+\sqrt{3}}$

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ب) $\frac{\sqrt{22}-1}{4+\sqrt{2}} + (2-\sqrt{3})^{-1} = \frac{2\sqrt{2}-1}{4+\sqrt{2}} + 2+\sqrt{3} = \frac{(2\sqrt{2}-1)(2-\sqrt{3})}{12} + 2+\sqrt{3} = \frac{4\sqrt{2}-2\sqrt{3}-2+2\sqrt{6}}{12} + 2+\sqrt{3}$

$\rightarrow \frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{1}$

الف) $\frac{\sqrt{1+\sqrt{3}} + \sqrt{3-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}$

$A = \sqrt{1+\sqrt{3}} + \sqrt{3-1}$
 $A^2 = 1+\sqrt{3} + 3-1 + 2\sqrt{2} = 3+\sqrt{3}+2\sqrt{2}$
 $A^2 = \sqrt{3} + \sqrt{2}$
 $A = \sqrt{2} \times \sqrt{\sqrt{3}+\sqrt{2}}$

$\rightarrow \frac{\sqrt{2} \times \sqrt{(\sqrt{3}+\sqrt{2})^2}}{1} = 2 = \sqrt{4} = 2$

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ب) $\sqrt[3]{\sqrt{2}} \times \sqrt[3]{142} \times \sqrt[3]{\sqrt{2}} = 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} = 2^2 = 4$

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$2^x + 2^{x+1} + 2^{x+2} + 2^{x+3} + 2^{x+4} + 2^{x+5} = 2^x (1+2+2^2+2^3+2^4+2^5) = 2^x (1+2+4+8+16+32) = 2^x (63)$
 $2^x (63) = 2^x \times 63 = 2^x \times 7 \times 9 = 2^x \times 7 \times 3^2 = 2^x \times 3^2 \times 7$
 $2^x \times 3^2 \times 7 = 2^x \times 9 \times 7 = 2^x \times 63$
 $2^x \times 63 = 2^x \times 63$
 $2^x \times 63 = 2^x \times 63$

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$\frac{2^x - 2^0}{2^x - 2^0} = 1$

$$a = \sqrt[3]{\sqrt{4}-2} \quad b = \sqrt[3]{\sqrt{4}+2}$$

$$(a^3+b^3-3ab)^3 (a^3+b^3+3ab)^3 = t(1-\sqrt{3})$$

$$(a^3-b^3)^3 = (a-b)^3 (a+b)^3 = t(1-\sqrt{3})$$

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$$A = (\sqrt[3]{\sqrt{4}-2})^3 - (\sqrt[3]{\sqrt{4}+2})^3$$

$$A = \sqrt{4}-2 - \sqrt{4}+2$$

$$A^3 = \sqrt{4}-2 + \sqrt{4}+2 - 2\sqrt{2}$$

$$A^3 = 2(\sqrt{4}-\sqrt{2}) = 2\sqrt{2}(\sqrt{2}-1)$$

$$A^3 = 2(2-2\sqrt{2}) = 4(1-\sqrt{2})$$

$$t = 1/4$$

$$a = \sqrt[3]{1-\sqrt{3}} = \sqrt[3]{(1-\sqrt{3})^3} = \sqrt{1-\sqrt{3}}$$

$$(a + \frac{1}{a} + \sqrt{3})^3 (a + \frac{1}{a} - \sqrt{3})^3 = 1^3$$

$$(a^3 + \frac{1}{a^3} + 3 - 3) = 1^3$$

$$(1-\sqrt{3} + 1 + \sqrt{3}) = 1^3 \Rightarrow t = 1$$

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$$A = \sqrt[3]{\sqrt[3]{19}} \left(\frac{1}{r}\right)^{\frac{1}{3}} \quad A = r^{\frac{1}{9}} \times r^{\frac{1}{9}} \times r^{\frac{1}{9}} = r^{\frac{1+1+1}{9}} = r^{\frac{3}{9}} = r^{\frac{1}{3}}$$

$$(rA)^{\frac{1}{3}} = (r^{\frac{1}{3}})^{\frac{1}{3}} = r^{-\frac{1}{3}} = \frac{1}{r}$$

(P) ✓

$$\sqrt[3]{a} = r \sqrt[3]{a}^{\frac{10}{3}} \quad a^{\frac{1}{3}} = r \sqrt[3]{a}^{\frac{10}{3}}$$

$$a^{\frac{1}{3}} = \frac{1}{r} \Rightarrow a = \frac{1}{\sqrt[3]{r}} = \frac{1}{r^{\frac{1}{3}}} = \frac{\sqrt[3]{r}}{r}$$

$$\frac{\frac{1}{a} - r}{1 + \sqrt{r}} = \frac{\frac{9}{\sqrt{r}} - r}{1 + \sqrt{r}} = \frac{3\sqrt{r} - r}{1 + \sqrt{r}} = \frac{3(\sqrt{r}-1)}{1 + \sqrt{r}} = \frac{3(\sqrt{r}-1)^2}{r}$$

$$\frac{3(r+1-2\sqrt{r})}{r} = \frac{4(r-\sqrt{r})}{4-3\sqrt{r}}$$

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$$A = \sqrt{x+a} - \sqrt{x-r} = r \quad B = \sqrt{x+a} + \sqrt{x-r}$$

$$AB = (\sqrt{x+a})^2 - (\sqrt{x-r})^2 = x+a - x+r = r+a$$

$$B = \frac{r+a}{r} = r + \frac{a}{r}$$

$$\sqrt{x+a} + \sqrt{x-r} - r = r + \frac{a}{r} - r = \frac{a}{r}$$

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