نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۱ کلاس .....

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$(1!+3!+5!+\cdots+25!)(2!+4!+6!+\cdots+26!)$

یکان: $1+6+0+\cdots+0=7$

یکان عبارت: $7\times 6 \to 42 \to \boxed{2}$

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الف) $\sqrt[4]{(4+\sqrt{7})^{-1}}\sqrt{1+\sqrt{7}} = \sqrt[4]{\frac{4-\sqrt{7}}{9}(1+\sqrt{7})^2} = \sqrt[4]{\frac{4-\sqrt{7}}{9}(8+2\sqrt{7})} = \sqrt[4]{\frac{32-16\sqrt{7}+16\sqrt{7}-112}{9}} = \sqrt[4]{4} = \boxed{\sqrt{2}}$

$\frac{1}{4+\sqrt{7}} = \frac{4-\sqrt{7}}{9}$

ب) $\frac{(\sqrt{2}+\sqrt{5})}{\sqrt{10}+2}\left(\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}}\right) = \frac{\sqrt{2}}{2}\times\sqrt{2} = 1$

$A = \sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}}$

$A^2 = 3-\sqrt{5}+3+\sqrt{5}-2\sqrt{9-5}$

$A^2 = 6-4 = 2 \quad A = -\sqrt{2}$

$\frac{(\sqrt{2}+\sqrt{5})(\sqrt{10}-2)}{4} = \frac{2\sqrt{5}-4\sqrt{2}+5\sqrt{2}-4\sqrt{5}}{4} = \frac{\sqrt{2}}{2}$

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الف) $\frac{\sqrt{11}+\sqrt{77}}{5-\sqrt{6}} - 2\left(\sqrt[3]{9}-1\right)^{-1} \to \frac{1}{\sqrt[3]{9}-1} = \frac{\sqrt[3]{3}+1}{2} \Rightarrow \sqrt{2}+\sqrt{3}-\sqrt{3}-1 = \sqrt{2}-1$

$\frac{(\sqrt{11}+3\sqrt{3})(5+\sqrt{6})}{19} = \frac{10\sqrt{2}+4\sqrt{3}+\sqrt{66}+9\sqrt{2}}{19} = \boxed{\sqrt{2}+\sqrt{3}}$

ب) $\frac{\sqrt{27}-1}{2+\sqrt{3}} + (2-\sqrt{3})^{-1} = \frac{3\sqrt{3}-1}{2+\sqrt{3}} + 2+\sqrt{3} = \frac{(3\sqrt{3}-1)(2-\sqrt{3})}{1} + 2+\sqrt{3} = 6\sqrt{3}-3-2+\sqrt{3} = \boxed{7\sqrt{3}-5}$

$\frac{1}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{1}$

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الف) $\frac{\sqrt{1+\sqrt{3}+\sqrt{\sqrt{3}-1}}}{\sqrt{\sqrt{2}-\sqrt{1}}} - 2$

$A = \sqrt{1+\sqrt{3}+\sqrt{\sqrt{3}-1}}$

$A^2 = 1+\sqrt{3}+\sqrt{3}-1+2\sqrt{1}$

$A^2 = 4\sqrt{3}+4\sqrt{1}$

$A = \sqrt{2}\times\sqrt{\sqrt{3}+\sqrt{1}}$

$\to \frac{\sqrt{2}\times\sqrt{(\sqrt{3}+\sqrt{1})^2}}{1} - 2 = \sqrt{2}(\sqrt{3}+\sqrt{1}) - 2 = \sqrt{6}+2-2 = \boxed{\sqrt{6}}$

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ب) $\sqrt[4]{\sqrt[3]{2^8}}\times\sqrt{\sqrt{64}}\times\sqrt{4\sqrt{2}} = 2^{\frac{8}{12}}\times 2^{\frac{6}{12}}\times 3\times 2\times 2^{\frac{1}{12}} = 3\times 2^{\frac{15}{12}} = \boxed{3\times 2^{\frac{5}{4}}}$

$64 = 4^3$

$\frac{3^x(1+3+9+27+81+243)}{2^{x-2}(1+2+4+8+16+32)} = 5^2$

$\frac{3^x(364)}{2^{x-2}(63)} = 5^2$

$\frac{3^x}{2^{x-2}} = 9$

$3^x = 9\times 2^{x-2}$

$3^{x-2} = 2^{x-2}$

$\frac{x-2=0}{x=2}$

$\frac{2^x+2^{x+1}+2^{x+2}+2^{x+3}+2^{x+4}+2^{x+5}}{2^{x-2}+2^{x-1}+2^x+2^{x+1}+2^{x+2}+2^{x+3}} = 5^2$

$2^{x+5}+2^{x+4}+2^{x+3}+2^{x+2}+2^{x+1}+2^x = 25\times 2^{x-2}+25\times 2^{x-1}+25\times 2^x+25\times 2^{x+1}+25\times 2^{x+2}+25\times 2^{x+3}$

$2^5\times 2^x+2^4\times 2^x+2^3\times 2^x+2^2\times 2^x+2\times 2^x+2^x = 25\times 2^{x-2}+25\times 2^{x-1}+25\times 2^x+25\times 2^{x+1}+25\times 2^{x+2}+25\times 2^{x+3}$

$2^x(32+16+8+4+2+1) = 25\times 2^{x-2}(1+2+4+8+16+32)$

$2^x\times 63 = 25\times 2^{x-2}\times 63$

$2^2\times 63 = 25\times 63$

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$a=\sqrt[3]{\sqrt{9}-2}\qquad b=\sqrt[3]{\sqrt{9}+2}$

$A=\left(\sqrt[3]{\sqrt{9}-2}\right)^3-\left(\sqrt[3]{\sqrt{9}+2}\right)^3$

$A=\sqrt{\sqrt{9}-2}-\sqrt{\sqrt{9}+2}$

$A^2=\sqrt{9-2}+\sqrt{9+2}-2\sqrt{2}$

$A^2=2\left(\sqrt{9}-\sqrt{2}\right)=2\sqrt{2}\left(\sqrt{2}-1\right)$

$A^3=4\left(2-2\sqrt{2}\right)=14\left(2-\sqrt{2}\right)$

$t=14$

$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2=t(2-\sqrt{2})$

$(a^2-b^2)^2=(a-b)^2(a+b)^2=t(2-\sqrt{2})$

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$a=\sqrt[3]{7-4\sqrt{2}}=\sqrt[3]{(2-\sqrt{2})^3}=\sqrt{2-\sqrt{2}}$

$\frac{1}{2-\sqrt{2}}=\frac{2+\sqrt{2}}{1}$

$\left(a+\frac{1}{a}+\sqrt{2}\right)^2\left(a+\frac{1}{a}-\sqrt{2}\right)^2=2^t$

$\left(a^2+\frac{1}{a^2}+2-2\right)^2=14=2^t\Rightarrow t=4$

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$A=\sqrt[3]{\sqrt[4]{16}}\left(\frac{1}{2}\right)^{-4}\qquad A=2^{\frac{1}{6}}\times 2^{\frac{1}{12}}\times 2^{\frac{1}{4}}=2^{\frac{2+1+3}{12}}=2^{\frac{6}{12}}=2^{\frac{1}{2}}$

$(2A)^{\frac{-1}{2}}=(2^2)^{\frac{-1}{2}}=2^{-1}=\frac{1}{2}$

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$\sqrt[7]{a}=128\,a^{\frac{15}{8}}\qquad a^{\frac{1}{8}}=128\,a^{\frac{15}{8}}\qquad a^4=\frac{1}{128}\Rightarrow a=\frac{1}{\sqrt{128}}=\frac{1}{8\sqrt{2}}=\frac{\sqrt{2}}{16}$

$\frac{\frac{1}{a}-3}{1+\sqrt{3}}=\frac{\frac{16}{\sqrt{2}}-3}{1+\sqrt{3}}=\frac{8\sqrt{2}-3}{1+\sqrt{3}}=\frac{3(\sqrt{3}-1)}{1+\sqrt{3}}=\frac{3(\sqrt{3}-1)^2}{2}=$

$\frac{3(4-2\sqrt{3}+1)}{2}=\frac{3(2-\sqrt{3})^2}{2}$

$8\sqrt{2}-6$

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$A=\sqrt{x+a}-\sqrt{x-2}=2\qquad B=\sqrt{x+a}+\sqrt{x-2}$

$AB=(\sqrt{x+a})^2-(\sqrt{x-2})^2=x+a-x+2=2+a$

$B=\frac{2+a}{2}=1+\frac{a}{2}$

$\sqrt{x+a}+\sqrt{x-2}-2=1+\frac{a}{2}-2=\frac{a}{2}$