نگارفتحی — بنام آفریدگار خورشید — دهم پنجشنبه دختر

۱- $(\underbrace{1!+3!}_{1+6=7}+\underbrace{5!+\dots+25!}_{\text{یکان}=0})(\underbrace{2!+4!+6!+\dots+26!}_{2+24=26\ \text{یکان}=0}) \Rightarrow 7\times 6=42$

لذا رقم یکان = ۲

۲- الف) $\sqrt[4]{\frac{1}{4+\sqrt{7}}}\times\sqrt[4]{(1+\sqrt{7})^2}=\sqrt[4]{\frac{1}{4+\sqrt{7}}\times\frac{(8+2\sqrt{7})}{2}}=\sqrt[4]{2}$

ب) $\frac{\sqrt{2+\sqrt{5}}}{\sqrt{2}(\sqrt{5}+\sqrt{2})}(\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}})=\frac{(\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}})}{\sqrt{2}\times\sqrt{2}}=\frac{\sqrt{2}}{2}(\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}})$

$\frac{\sqrt{6-2\sqrt{5}}}{2}-\frac{\sqrt{6+2\sqrt{5}}}{2}=\xrightarrow{\sqrt{2}\times\sqrt{2}}\frac{(\sqrt{5}-1)-(\sqrt{5}+1)}{2}=-1$

۳- الف) $\frac{(2\sqrt{2}+3\sqrt{3})(5+\sqrt{6})}{(5-\sqrt{6})(5+\sqrt{6})}=\frac{10\sqrt{2}+6\sqrt{3}+15\sqrt{3}+9\sqrt{2}}{25-6}=\frac{19(\sqrt{2}+\sqrt{3})}{19}=\sqrt{2}+\sqrt{3}$

$2(\sqrt{9}-1)^{-1}\xrightarrow{}2(\sqrt{3}-1)^{-1}=\frac{2(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}=\frac{2(\sqrt{3}+1)}{2}=\sqrt{3}+1$ $\quad \sqrt{2}+\sqrt{3}-(\sqrt{3}-1)$

$3-1=2$

لذا $\sqrt{2}-1$ جواب

ب) $\frac{(3\sqrt{3}-1)(4-\sqrt{3})}{(4+\sqrt{3})(4-\sqrt{3})}=\frac{(3\sqrt{3}-1)(4-\sqrt{3})}{16-3}=\frac{12\sqrt{3}-9-4+\sqrt{3}}{13}=\frac{13(\sqrt{3}-1)}{13}\longrightarrow\sqrt{3}-1$

$(2-\sqrt{3})^{-1}=\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{2+\sqrt{3}}{4-3}=2+\sqrt{3}\longrightarrow\sqrt{3}-1+2+\sqrt{3}=1+2\sqrt{3}$

۴- الف) $\left(\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}\right)^2=\frac{1+\sqrt{3}+\sqrt{3}-1+2\sqrt{(\sqrt{3}+1)(\sqrt{3}-1)}}{\sqrt{3}-\sqrt{2}}=\frac{2\sqrt{3}+2\sqrt{3-1}}{\sqrt{3}-\sqrt{2}}=\frac{2(\sqrt{2}+\sqrt{3})}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=$

$3-2=1$

$\frac{2(\sqrt{2}+\sqrt{3})^2}{1}=2+3+2\sqrt{6}=5+2\sqrt{6}$

$\sqrt{5+2\sqrt{6}}=\sqrt{(2+\sqrt{6})^2}=2+\sqrt{6}\longrightarrow 2+\sqrt{6}-2=\sqrt{6}$

ب) $2\frac{8}{12}\Rightarrow 2\frac{2}{3}\times 3\times 2\frac{1}{2}\times 2\times 2\frac{1}{12}=2\frac{12}{12}\times 6=12$

$$\frac{3^{x}(1+3+9+27+81+243)}{2^{x-2}(1+2+4+8+16+32)} = 5 \longrightarrow \frac{3^{x-2}}{2^{x-2}} = 1 \Rightarrow \left(\frac{3}{2}\right)^{x-2} = \left(\frac{3}{2}\right)^{x} \times \frac{4}{9} = 1-5$$

$$\left(\frac{3}{2}\right)^{x} = \frac{9}{4} \longrightarrow x=2$$

-6

$$((a-b)^2)^2((a+b)^2)^2 = (a-b)^4(a+b)^4 \longrightarrow ((a-b)(a+b))^4$$

$$((a^2-b^2)^4)$$

$$\left(\left(\sqrt{\sqrt{6}+2} - \sqrt{\sqrt{6}-2}\right)^2\right)^4 = \left(\sqrt{6}+2+\sqrt{6}-2-2\sqrt{\sqrt{6}-4}\right)^2 \longrightarrow (2\sqrt{6}-2\sqrt{2})^2$$

$$24+8-16\sqrt{3} = 32-16\sqrt{3} = 16(2-\sqrt{3}) \xrightarrow{2\sqrt{2}} t(2-\sqrt{3}) = 16(2-\sqrt{3})$$

$$\longrightarrow t=16$$

-7

$$a = \sqrt[4]{7-4\sqrt{3}} = \sqrt{2-\sqrt{3}} \qquad \left(\left(a+\frac{1}{a}\right)^2 - (\sqrt{2})^2\right)^2 = \left(a^2+\frac{1}{a^2}+2-2\right)^2$$

$$\frac{1}{a^2} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2+\sqrt{3} \longrightarrow (2+\sqrt{3}+2-\sqrt{3})^2 \longrightarrow 4^2 = 16 \longrightarrow t=4$$

$$4-3=1 \qquad \longrightarrow 2^4 = t^4$$

-8

$$A = 2^{\frac{2}{5}} \times 2^{\frac{4}{15}} \times 2^{\frac{4}{3}} = 2^{\frac{20}{15}} \times 2^{\frac{2}{5}} = 2^2 = 4 \qquad (2A)^{-\frac{1}{2}} = (2(2^3))^{-\frac{1}{2}} = \frac{1}{2}$$

$$2^3$$

-9

$$\sqrt[7]{a} = 27a^{\frac{15}{2}} \xrightarrow{\text{به توان } 7} (\sqrt[7]{a})^7 = (3^3)^7 \times a^{\frac{15}{2}\times 7} = a = 3^{21} \times a^{\frac{105}{2}} \Rightarrow a = 3^{21} \longrightarrow a^{-1} = 3^{-\frac{21}{14}} = 9$$

$$a^{-1} = 3\sqrt{3} \longrightarrow \frac{1}{a} - 3 = 3\sqrt{3} - 3 = 3(\sqrt{3}-1) \longrightarrow \frac{3(\sqrt{3}-1)}{1+\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3(3+1+2\sqrt{3})}{2}$$

$$3-1=2 \qquad \frac{12+6\sqrt{3}}{2} = 2+3\sqrt{3}$$

-10

$$(\sqrt{x+a} - \sqrt{x-4})(\sqrt{x+a} + \sqrt{x-4}) \Rightarrow x+a-(x-4) = a+4$$

$$\sqrt{x+a} + \sqrt{x-4} = \frac{a+4}{2} \longrightarrow \frac{a+4}{2} - 2 \Rightarrow \frac{a+4-4}{2} = \frac{a}{2}$$