بنام خدا

روزشنبه خنه‌ای

۲۰

۱ / ۱ / ۷

① $(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+26!)$

$1!=1,\ 3!=6,\ 5!=120 \Rightarrow 7$; $2!\to 2,\ 4!\to 24,\ 6!\to 720 \Rightarrow 6$; $7\times 6 = 42$ — یکان ۲

ص

②

الف) $\sqrt[4]{(4+\sqrt{7})^{-1}}\sqrt{1+\sqrt{7}} \Rightarrow \sqrt[4]{(4+\sqrt{7})^{-1}(1+\sqrt{7})^{2}} \Rightarrow \sqrt[4]{\frac{1+7+2\sqrt{7}}{4+\sqrt{7}}}$

$\sqrt[4]{\frac{8+2\sqrt{7}}{4+\sqrt{7}}} = \sqrt[4]{\frac{2\cancel{(4+\sqrt{7})}}{\cancel{4+\sqrt{7}}}} = \boxed{\sqrt[4]{2}}$

ب) $\underbrace{\left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+3}\right)}_{A}\underbrace{\left(\underbrace{\sqrt{3-\sqrt{5}}}_{x}-\underbrace{\sqrt{3+\sqrt{5}}}_{y}\right)}_{B} \Rightarrow A \Rightarrow \frac{\cancel{\sqrt{2}+\sqrt{5}}}{\sqrt{2}\cancel{(\sqrt{2}+\sqrt{5})}} = \frac{1}{\sqrt{2}}$

$(x-y)^2 \Rightarrow 3-\cancel{\sqrt{5}}+3+\cancel{\sqrt{5}}-2\sqrt{(3-\sqrt{5})(3+\sqrt{5})}$; $9-5=4$

$6-4=2 \Rightarrow x-y=-\sqrt{2}$ ص $\Rightarrow AB \Rightarrow \frac{1}{\sqrt{2}}\times-\sqrt{2}=\boxed{-1}$

③

الف) $\frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{6}}-2\left(\sqrt[4]{9}-1\right)^{-1} \Rightarrow \frac{2\sqrt{2}+3\sqrt{3}}{5-\sqrt{6}}\times\frac{5+\sqrt{6}}{5+\sqrt{6}} \Rightarrow$

$\frac{\overbrace{10\sqrt{2}+9\sqrt{2}}^{19\sqrt{2}}+\overbrace{15\sqrt{3}+4\sqrt{3}}^{19\sqrt{3}}}{19}=\sqrt{2}+\sqrt{3}$

$\frac{2}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{\cancel{2}(\sqrt{3}+1)}{\cancel{2}}$

$\sqrt{2}+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}-1=\boxed{\sqrt{2}-1}$

ب) $\underbrace{\frac{3\sqrt{3}-1}{4+\sqrt{3}}}_{A}+\underbrace{(2-\sqrt{3})^{-1}}_{B} \Rightarrow$ ص

$A=\frac{3\sqrt{3}-1}{4+\sqrt{3}}\times\frac{4-\sqrt{3}}{4-\sqrt{3}} \Rightarrow \frac{12\sqrt{3}-4-9+\sqrt{3}}{13} \Rightarrow \frac{13\sqrt{3}-13}{13}=\sqrt{3}-1$

$B=\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}} \Rightarrow \frac{2+\sqrt{3}}{4-3}=2+\sqrt{3}$

$\sqrt{3}-1+2+\sqrt{3} \Rightarrow \boxed{2\sqrt{3}+1}$

صورت سوال ۲

الف)

۴)

$$\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}-2 \Rightarrow 1+\sqrt{3}+\sqrt{3}-1+2\left(\sqrt{3-1}\right) \Rightarrow$$

$$2\sqrt{3}+2\sqrt{2} \xrightarrow{\text{ریشه گرفتن}} \frac{\sqrt{2\left(\sqrt{3}+\sqrt{2}\right)}}{\sqrt{\sqrt{3}-\sqrt{2}}} \Rightarrow$$

$$\sqrt{\frac{2(\sqrt{3}+\sqrt{2})}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}} \Rightarrow \sqrt{2(\underbrace{3+2}_{5}+2\sqrt{6})} \Rightarrow \sqrt{\underbrace{10+4\sqrt{6}}_{2\times 2\sqrt{6}}} \Rightarrow \sqrt{(\sqrt{6}+2)^2}=\sqrt{6}+2$$

$$\sqrt{6}+2-2=\boxed{\sqrt{6}}$$

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ب)

$$\sqrt[4]{\sqrt[3]{2^8}}\times\sqrt[4]{162}\times\sqrt{4\sqrt[6]{2}} \Rightarrow 2^{\cancel{8}^{2}\times\frac{1}{3}\times\frac{1}{\cancel{4}}}\times 162^{\frac{1}{4}}\times 4^{\frac{1}{2}}\times 2^{\frac{1}{6}}$$

$$2^{\cancel{8}\times\frac{1}{3}\times\frac{1}{\cancel{4}}}\times 2^{\frac{1}{4}}\times 2\times \cancel{81}^{\,3^4\times\frac{1}{4}}\times 2^{\frac{1}{6}}\times 3 \rightarrow 2^{\frac{2}{3}+\frac{1}{4}+\frac{1}{12}+1}\times 3 \Rightarrow 2^{\frac{8+3+1+12}{12}}\times 3$$

$$2^2\times 3 \rightarrow \boxed{12}$$

۵)

$$\frac{3^x+3^{x+1}+3^{x+2}+3^{x+3}+3^{x+4}+3^{x+5}}{2^{x-2}+2^{x-1}+2^x+2^{x+1}+2^{x+2}+2^{x+3}}=52$$

$$\frac{3^x(1+\cancel{3}+\cancel{9}+\cancel{27}+\cancel{81}+243)^{364}}{2^{x-2}(1+\cancel{2}+\cancel{4}+\cancel{8}+\cancel{16}+32)_{63}}=52 \Rightarrow \frac{3^x}{2^{x-2}}=\frac{\cancel{63}\times\cancel{52}}{\cancel{364}}^{9}$$

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$$\frac{3^x}{2^{x-2}}=9 \rightarrow \frac{3^x}{2^{x-2}}=3^2 \Rightarrow x=2$$

۶)

$$a=\sqrt[4]{\sqrt{6}-2}$$
$$b=\sqrt[4]{\sqrt{6}+2}$$

$$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2=4(2-\sqrt{3})$$

$$\left(\sqrt{\sqrt{6}-2}+\sqrt{\sqrt{6}+2}-2\sqrt{2}\right)\left(\sqrt{\sqrt{6}-2}+\sqrt{\sqrt{6}+2}+2\sqrt{2}\right)$$

$$AB=\sqrt[4]{(\sqrt{6}-2)(\sqrt{6}+2)} \Rightarrow \sqrt[4]{6-4}=\sqrt[4]{2} \rightarrow 2AB=2\sqrt[4]{2}$$

$$\boxed{16}$$

$$\xrightarrow{\text{داریم}} \sqrt{6}-\cancel{2}+\sqrt{6}+\cancel{2}-2\sqrt{\frac{6-4}{2\sqrt{2}}}-4\sqrt{2}$$

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$$4(8-2\sqrt{12})=16(2-\sqrt{3})$$

$$2\sqrt{6}-2\sqrt{2}-4\sqrt{2} \rightarrow$$

$$2(\sqrt{6}-\sqrt{2})^2 \Rightarrow 4(\sqrt{6}-\sqrt{2})^2 \Rightarrow 4(6+2-\sqrt{12}) \rightarrow$$

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$a=\sqrt[۴]{۷-\underbrace{۴\sqrt{۳}}_{۲\times ۲\sqrt{۳}}} \longrightarrow a=\sqrt[۴]{(۲-\sqrt{۳})^۲}=\sqrt[۲]{(۲-\sqrt{۳})}$

$(\underbrace{a+\frac{۱}{a}}_{A}+\sqrt{۲})(a+\frac{۱}{a}-\underbrace{\sqrt{۲}}_{B})^{۲} \Rightarrow (a^۲+\frac{۱}{a^۲})+\cancel{۲}-\cancel{۲} \rightarrow a^۲+\frac{۱}{a^۲}+۲$

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$(۲-\sqrt{۳})^۲+\frac{۱}{(۲-\sqrt{۳})^۲}+۲=۲^t \rightarrow \frac{۱}{۷-۴\sqrt{۳}}\times\frac{۷+۴\sqrt{۳}}{۷+۴\sqrt{۳}} \Rightarrow$

$\frac{۷+۴\sqrt{۳}}{۴۹-۴۸}=۷+۴\sqrt{۳} \longrightarrow ۷-\cancel{۴\sqrt{۳}}+۷+\cancel{۴\sqrt{۳}}+۲=۲^t$

$۱۶=۲^t \rightarrow t=۴$

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$A=\sqrt[۵]{۴\sqrt[۳]{۱۶}}\left(\frac{۱}{۲}\right)^{-\frac{۲}{۳}} \Longrightarrow$

$۲^{۴\times\frac{۱}{۳}\times\frac{۱}{۵}}\times ۲^{۲\times\frac{۱}{۵}}\times ۲^{\frac{۴}{۳}} \Rightarrow ۲^{\frac{۴}{۱۵}+\frac{۲}{۵}+\frac{۴}{۳}} \Rightarrow ۲^{\frac{۴+۶+۲۰}{۱۵}}=۲^۲=\boxed{۴}$

$(۲A)^{-\frac{۱}{۳}} \longrightarrow (۲\times ۴)^{-\frac{۱}{۳}} \longrightarrow ۸^{-\frac{۱}{۳}} \longrightarrow \frac{۱}{۸}^{\frac{۱}{۳}}=\boxed{\sqrt[۳]{\frac{۱}{۸}}}=\boxed{\frac{۱}{۲}}$

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$a^{\frac{۱}{۷}}=\underbrace{۲۷}_{۳^۳}\times a^{\frac{۱۵}{۷}} \longrightarrow a^{-\frac{۱۴}{۷}} \rightarrow a^{-۲}=۳^۳$

$\left(\frac{۱}{a}\right)^{۲}=۳^۳$

$\frac{۱}{a}=\sqrt[۲]{۲۷}$

$\rightarrow \frac{۱}{a}-۳=۳\sqrt{۳}-۳$

$\frac{۳\sqrt{۳}-۳}{۱+\sqrt{۳}}\times\frac{۱-\sqrt{۳}}{۱-\sqrt{۳}} \Rightarrow \frac{۳\sqrt{۳}-۹-۳+۳\sqrt{۳}}{\cancel{۱-۳}\ -۲} \Rightarrow \frac{۶\sqrt{۳}-۱۲}{-۲}=-۳\sqrt{۳}+۶$

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$\sqrt{\frac{x+a}{۴}}-\sqrt{\frac{x-۴}{۴}}=۱ \rightarrow \sqrt{x+a}-\sqrt{x-۴}=۲$

$\sqrt{x+a}-\sqrt{x-۴}=۲ \longrightarrow (t+w)(t-w) \Rightarrow t^۲-w^۲ \Rightarrow$

$t-w=۲ \rightarrow$

$x+a-x+۴=a+۴$

$\frac{a+۴}{۲} \Rightarrow t+w-۲ \Rightarrow \frac{a+۴-۴}{۲}$

۲ ✓

$\boxed{\frac{a}{۲}}$