روزشمار خدای   بنام خدا

۱ / ۱ / ۷

(۱

$(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+26!)$

$1! = 1$, $3! = 6$, $5! = 120$ → ۷

$2! \to 2$, $4! \to 24$, $6! \to 720$ → ۶

$7 \times 6 = 42$ — گزینه ۲

(۲

الف) $\sqrt[4]{(4+\sqrt{7})^{-1}}\sqrt{1+\sqrt{7}} \Rightarrow \sqrt[4]{(4+\sqrt{7})^{-1}(1+\sqrt{7})^2} \Rightarrow \sqrt[4]{\dfrac{1+7+2\sqrt{7}}{4+\sqrt{7}}}$

$\sqrt[4]{\dfrac{8+2\sqrt{7}}{4+\sqrt{7}}} = \sqrt[4]{\dfrac{2(\cancel{4+\sqrt{7}})}{\cancel{4+\sqrt{7}}}} = \boxed{\sqrt[4]{2}}$

ب) $\underbrace{\left(\dfrac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2}\right)}_{A}\left(\underbrace{\sqrt{3-\sqrt{5}}}_{x}\ \underbrace{-}_{B}\ \underbrace{\sqrt{3+\sqrt{5}}}_{y}\right) \Rightarrow A \Rightarrow \dfrac{\cancel{\sqrt{2}+\sqrt{5}}}{\sqrt{2}(\cancel{\sqrt{2}+\sqrt{5}})} = \dfrac{1}{\sqrt{2}}$

$(x-y)^2 \Rightarrow 3-\cancel{\sqrt{5}}+3+\cancel{\sqrt{5}}-2\sqrt{(3-\sqrt{5})(3+\sqrt{5})}$

$9-5=4 \to 2$

$6-4=2 \to x-y=-\sqrt{2} \Rightarrow AB \Rightarrow \dfrac{1}{\sqrt{2}}\times -\sqrt{2} = \boxed{-1}$

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الف) $\dfrac{\sqrt{8}+\sqrt{27}}{5-\sqrt{6}} - 2(\sqrt[4]{9}-1)^{-1} \Rightarrow \dfrac{2\sqrt{2}+3\sqrt{3}}{5-\sqrt{6}} \dfrac{\times 5+\sqrt{6}}{\times 5+\sqrt{6}} \Rightarrow$

$\dfrac{\overbrace{10\sqrt{2}+9\sqrt{2}}^{19\sqrt{2}}+\overbrace{15\sqrt{3}+4\sqrt{3}}^{19\sqrt{3}}}{19} = \sqrt{2}+\sqrt{3}$

$\dfrac{2}{\sqrt[2]{3}-1} \dfrac{\times\sqrt{3}+1}{\times\sqrt{3}+1} = \dfrac{\cancel{2}(\sqrt{3}+1)}{\cancel{2}}$

$\sqrt{2}+\sqrt{3}-\cancel{\sqrt{3}}-1 = \boxed{\sqrt{2}-1}$

ب) $\underbrace{\dfrac{3\sqrt{3}-1}{4+\sqrt{3}}}_{A} + \underbrace{(2-\sqrt{3})^{-1}}_{B} \Rightarrow$

$A = \dfrac{3\sqrt{3}-1}{4+\sqrt{3}} \dfrac{\times 4-\sqrt{3}}{\times 4-\sqrt{3}} \Rightarrow \dfrac{12\sqrt{3}-4-9+\sqrt{3}}{13} \Rightarrow \dfrac{13\sqrt{3}-13}{13} = \sqrt{3}-1$

$B = \dfrac{1}{2-\sqrt{3}} \dfrac{\times 2+\sqrt{3}}{\times 2+\sqrt{3}} \Rightarrow \dfrac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$

$\sqrt{3}-1+2+\sqrt{3} \Rightarrow \boxed{2\sqrt{3}+1}$

صورت سوال ۲

(۴ الف)

$$\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}-2 \Rightarrow 1+\sqrt{3}+\sqrt{3}-1+2(\sqrt{3-1}) \Rightarrow$$

$$2\sqrt{3}+2\sqrt{2} \xrightarrow{\text{در صورت}} \frac{\sqrt{2(\sqrt{3}+\sqrt{2})}}{\sqrt{\sqrt{3}-\sqrt{2}}} \Rightarrow$$

$$\sqrt{\frac{2(\sqrt{3}+\sqrt{2})}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}} \Rightarrow \sqrt{2(\underbrace{3+2}_{5}+2\sqrt{6})} \Rightarrow \sqrt{10+\underbrace{4\sqrt{6}}_{2\times 2\sqrt{6}}} \Rightarrow \sqrt{(\sqrt{6}+2)^2}=\sqrt{6}+2$$

$$\sqrt{6}+2-2=\boxed{\sqrt{6}}$$

ب)

$$\sqrt[4]{\sqrt[3]{2^8}}\times\sqrt[4]{162}\times\sqrt{4\sqrt[6]{2}} \Rightarrow 2^{\cancel{8}^{2}\times\frac{1}{3}\times\frac{1}{\cancel{4}}}\times 162^{\frac{1}{4}}\times 4^{\frac{1}{2}}\times 2^{\frac{1}{6}}$$

$$2^{\cancel{8}\times\frac{1}{3}\times\frac{1}{\cancel{4}}}\times 2^{\frac{1}{4}}\times 2\times 3^{\cancel{4}\times\frac{1}{4}}\times 2^{\frac{1}{6}\times\frac{1}{2}}\times 3 \to 2^{\frac{2}{3}+\frac{1}{4}+\frac{1}{12}+1}\times 3 \Rightarrow 2^{\frac{8+3+1+12}{12}}\times 3$$

$$2^2\times 3\to\boxed{12}$$

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$$\frac{3^x+3^{x+1}+3^{x+2}+3^{x+3}+3^{x+4}+3^{x+5}}{2^{x-2}+2^{x-1}+2^{x}+2^{x+1}+2^{x+2}+2^{x+3}}=52$$

$$\frac{3^x(\overbrace{1+\cancel{3}+\cancel{9}+\cancel{27}+\cancel{81}+243}^{364})}{2^{x-2}(\underbrace{1+\cancel{2}+\cancel{4}+\cancel{8}+\cancel{16}+32}_{63})}=52 \Rightarrow \frac{3^x}{2^{x-2}}=\frac{\cancel{63}^{9}\times\cancel{52}}{\cancel{364}}$$

$$\frac{3^x}{2^{x-2}}=9 \to \frac{3^x}{2^{x-2}}=3^2 \Rightarrow \boxed{x=2}$$

(۶)

$$a=\sqrt[4]{\sqrt{6}-2}$$
$$b=\sqrt[4]{\sqrt{6}+2}$$

$$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2=?(2-\sqrt{3})$$

$$(\sqrt{\sqrt{6}-2}+\sqrt{\sqrt{6}+2}-2\sqrt[4]{2})(\sqrt{\sqrt{6}-2}+\sqrt{\sqrt{6}+2}+2\sqrt[4]{2})$$

$$AB=\sqrt[4]{(\sqrt{6}-2)(\sqrt{6}+2)}\Rightarrow\sqrt[4]{6-4}=\sqrt[4]{2}\to 2AB=2\sqrt[4]{2}$$

$$\boxed{=16}$$

$$\xrightarrow{\text{با}} \sqrt{6}-\cancel{2}+\sqrt{6}+\cancel{2}-\frac{2\sqrt{6-4}}{2\sqrt{2}}-4\sqrt{2}$$

$$4(8-2\sqrt{12})=16(2-\sqrt{3})$$

$$2\sqrt{6}-2\sqrt{2}-4\sqrt{2}\to$$

$$2(\sqrt{6}-\sqrt{2})^2\Rightarrow 4(\sqrt{6}-\sqrt{2})^2\Rightarrow 4(6+2-\sqrt{12})\to$$

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(۷)

$a = \sqrt[4]{\underbrace{7-4\sqrt{3}}} \longrightarrow a = \sqrt[4]{(2-\sqrt{3})^2} = \sqrt[2]{(2-\sqrt{3})}$

$32 \quad 2\sqrt{3}$

$(\underbrace{a+\frac{1}{a}}_{A}+\sqrt{2})(\underbrace{a+\frac{1}{a}}-\underbrace{\sqrt{2}}_{B})^{2} \Rightarrow (a^2+\frac{1}{a^2})^{2}+\cancel{2}-\cancel{2} \to a^4+\frac{1}{a^4}+2$

$(2-\sqrt{3})^2 + \frac{1}{(2-\sqrt{3})^2} + 2 = 2^t \longrightarrow \frac{1}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} \Rightarrow$

$\frac{7+4\sqrt{3}}{49-48} = 7+4\sqrt{3} \longrightarrow 7-\cancel{4\sqrt{3}}+7+\cancel{4\sqrt{3}}+2 = 2^t$

$16 = 2^t \longrightarrow t = 4$

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$A = \sqrt[5]{4\sqrt[3]{16}}\,(\frac{1}{2})^{-\frac{2}{3}} \Longrightarrow$

$2^{4\times\frac{1}{3}\times\frac{1}{5}} \times 2^{2\times\frac{1}{5}} \times 2^{\frac{2}{3}} \Rightarrow 2^{\frac{4}{15}+\frac{2}{5}+\frac{2}{3}} \Rightarrow 2^{\frac{4+6+10}{15}} = 2^2 = \boxed{4}$

$(2A)^{-\frac{1}{3}} \longrightarrow (2\times 4)^{-\frac{1}{3}} \longrightarrow 8^{-\frac{1}{3}} \longrightarrow \frac{1}{8}^{\frac{1}{3}} = \boxed{\sqrt[3]{\frac{1}{8}}} = \boxed{\frac{1}{2}}$

(۹)

$a^{\frac{1}{7}} = 27 \times a^{\frac{15}{7}} \longrightarrow a^{-\frac{14}{7}} \to a^{-2} = 3^3$

3^3

$(\frac{1}{a})^2 = 3^3$

$\frac{1}{a} = \sqrt[2]{27}$

$\to \frac{1}{a} - 3 = 3\sqrt{3} - 3$

$\frac{3\sqrt{3}-3}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}} \Rightarrow \frac{3\sqrt{3}-9-3+3\sqrt{3}}{\cancel{1-3}\ -2} \Rightarrow \frac{6\sqrt{3}-12}{-2} = -3\sqrt{3}+6$

(۱۰)

$\sqrt{\frac{x+a}{4}} - \sqrt{\frac{x-4}{4}} = 2 \to \sqrt{x+a} = \frac{\sqrt{x-4}+2}{2}$

$\sqrt{x+a} + \sqrt{x-4} = 2 \longrightarrow (t+w)(t-w) \Rightarrow t^2 - w^2 \Rightarrow$

$t+w = 2 \to \qquad x+a-x+4 = a+4$

dotnote

$\frac{a+4}{2} \Rightarrow t+w=2 \Rightarrow \frac{a+4-4}{2}$

$\boxed{\frac{a}{2}}$