آرمینا کبیری گروه دهم پنجشنبه C

۲۰

(۱)

$(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+26!)$

$1+6=7$ — یکان صفرات — $2+24=26$ — یکان صفرات

$\rightarrow 7\times 6=4②$ یکان ۲ است

② ✓

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(۲)

الف) $\sqrt[4]{\frac{1}{4+\sqrt7}}\times\sqrt[4]{(1+\sqrt7)^2}=\sqrt[4]{\frac{1}{4+\sqrt7}\times(8+2\sqrt7)}=\sqrt[4]{2}$

ب) $\frac{\sqrt5+\sqrt2}{\sqrt2(\sqrt5+\sqrt2)}\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)=\frac{\sqrt2}{2}\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)$

$=\frac{\sqrt{6-2\sqrt5}}{2}-\frac{\sqrt{6+2\sqrt5}}{2}=\frac{|\sqrt5-1|-(\sqrt5+1)}{2}=(-1)$

② ✓

subject

Year Month Date ( )

(۲)

الف) عبارت اول $\rightarrow \frac{2\sqrt{2}+3\sqrt{3}}{5-\sqrt{6}} \times \frac{5+\sqrt{6}}{5+\sqrt{6}} = \frac{10\sqrt{2}+6\sqrt{3}+15\sqrt{3}+9\sqrt{2}}{19} = \frac{19(\sqrt{2}+\sqrt{3})}{19} = \sqrt{2}+\sqrt{3}$

عبارت دوم $\rightarrow 2(\sqrt[4]{9}-1)^{-1} = 2(\sqrt{3}-1)^{-1} = \frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{2(\sqrt{3}+1)}{2} = \sqrt{3}+1$

جواب کلی $\rightarrow \sqrt{2}+\sqrt{3}-\sqrt{3}-1 = \boxed{\sqrt{2}-1}$ ۲ ✓

ب) عبارت اول $\rightarrow \frac{3\sqrt{3}-1}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{(3\sqrt{3}-1)(4-\sqrt{3})}{13} = \frac{13\sqrt{3}-13}{13} = \frac{13(\sqrt{3}-1)}{13} = \sqrt{3}-1$

عبارت دوم $\rightarrow (2-\sqrt{3})^{-1} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$ ✓

جواب کلی $\rightarrow \sqrt{3}-1+2+\sqrt{3} = \boxed{1+2\sqrt{3}}$

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(۳)

الف) $\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} = A \rightarrow A^2 = \frac{1+\sqrt{3}+\sqrt{3}-1+2\sqrt{(\sqrt{3}+1)(\sqrt{3}-1)}}{\sqrt{3}-\sqrt{2}} = \frac{2(\sqrt{3}+\sqrt{2})}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \frac{2(\sqrt{3}+\sqrt{2})^2}{1} = 10+4\sqrt{6} \rightarrow A = \sqrt{10+4\sqrt{6}} = 2+\sqrt{6}$

$2+\sqrt{6}-2 = \sqrt{6}$ ۲ ✓

ب) $2^{\frac{2}{3}} \times 3 \times 2^{\frac{1}{4}} \times 2 \times 2^{\frac{1}{12}} = 2^2 \times 3 = 12$

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(۴)

$\frac{3^n(1+3+9+27+81+243)}{2^{n-2}(1+2+4+8+16+32)} = 54$

$\frac{3^{n-2}}{2^{n-2}} = 1 \rightarrow (\frac{3}{2})^n \times \frac{4}{9} = 1$

$\rightarrow (\frac{3}{2})^n = \frac{9}{4} \rightarrow \boxed{n=2}$ ۲ ✓

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(۵)

$(a+b)^4(b-a)^4 = ((b+a)(b-a))^4 = (b^2-a^2)^4$

$= |(\sqrt{\sqrt{6}+2}-\sqrt{\sqrt{6}-2})^2|^2 = (\sqrt{6}+2+\sqrt{6}-2-2\sqrt{6-4})^2 = (2\sqrt{6}-2\sqrt{2})^2$

$= 24+8-16\sqrt{3} = 32-16\sqrt{3} = 16(2-\sqrt{3}) \rightarrow \boxed{t=16}$ ۲ ✓

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(۶)

$a = \sqrt[4]{(2-\sqrt{3})^2} = \sqrt{2-\sqrt{3}} \rightarrow \frac{1}{a^2} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2+\sqrt{3}$

$((a+\frac{1}{a})^2-2)^2 = (a^2+\frac{1}{a^2}+2-2)^2 = (a^2+\frac{1}{a^2})^2 = (2-\sqrt{3}+2+\sqrt{3})^2$

$= 4^2 = 16 \rightarrow \boxed{t=4}$ ۲ ✓

Parsian

$A = 2^{\frac{2}{5}} \times 2^{\frac{4}{15}} \times 2^{\frac{4}{3}} = 2^{2} = 4 \rightarrow (2A)^{-\frac{1}{3}} = (2^{3})^{-\frac{1}{3}} = \frac{1}{2}$ (۸)

$\sqrt[7]{a} = 2\sqrt{a}\, a^{\frac{15}{7}} \longrightarrow a = 3^{21} \times a^{15} \rightarrow a^{-14} = 3^{21} \rightarrow a^{-1} = 3\sqrt{3}$ (۹)

$\frac{1}{a} - 3 = 3(\sqrt{3}-1) \qquad \frac{3(\sqrt{3}-1)}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{12-6\sqrt{3}}{2} = 6-3\sqrt{3}$

(۵) ✓

$\left(\frac{\sqrt{n+a} - \sqrt{n-4}}{2}\right)\left(\frac{\sqrt{n+a} + \sqrt{n-4}}{\frac{a+4}{2}}\right) = n+a-(n-4) = a+4$ (۱۰)

(۲)

$\frac{a+4}{2} - 2 = \frac{a}{2}$

✓

Parsian