پرنیان دهقانی

۱ـ

$$(۱!+۲!+۳!+۴!+\underbrace{۵!+\dots}_{\text{رقم یکان صفر}})(۲!+۴!+\underbrace{۶!+\dots}_{\text{رقم یکان صفر}}) \Rightarrow (\underbrace{۱+۲+۶+۲۴}_{۳۳})(\underbrace{۲+۲۴}_{۲۶})$$

$$۶\times ۳ = ۸$$

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۲ـ

الف)

$$\sqrt[۴]{(۴+\sqrt{۷})^{-۱}} \times \sqrt{۱+\sqrt{۷}} = (۴+\sqrt{۷})^{-\frac{۱}{۴}}(۱+\sqrt{۷})^{\frac{۱}{۲}}$$

$$= (۱+\sqrt{۷})^{\frac{۲}{۴}}(۴+\sqrt{۷})^{-\frac{۱}{۴}} = \left(\frac{(۱+\sqrt{۷})^۲}{(۴+\sqrt{۷})}\right)^{\frac{۱}{۴}} \Rightarrow \left(\frac{۱+۷+۲\sqrt{۷}}{۴+\sqrt{۷}}\right)^{\frac{۱}{۴}} = \left(\frac{۸+۲\sqrt{۷}}{۴+\sqrt{۷}}\right)^{\frac{۱}{۴}} = \left(\frac{۲(\cancel{۴+\sqrt{۷}})}{\cancel{۴+\sqrt{۷}}}\right)^{\frac{۱}{۴}} = (۲)^{\frac{۱}{۴}} = \sqrt[۴]{۲}$$

ب)

$$\frac{\sqrt{۲}+\sqrt{۵}}{\sqrt{۱۰}+۲} = \frac{\cancel{\sqrt{۲}+\sqrt{۵}}}{\sqrt{۲}(\cancel{\sqrt{۵}+\sqrt{۲}})} = \frac{۱}{\sqrt{۲}} = \frac{\sqrt{۲}}{۲} \Big/ \left(\sqrt{۳-\sqrt{۵}} - \sqrt{۳+\sqrt{۵}}\right) = \left(\sqrt{\frac{۵}{۲}} - \sqrt{\frac{۱}{۲}}\right) - \left(\sqrt{\frac{۵}{۲}} + \sqrt{\frac{۱}{۲}}\right)$$

$$= -۲\sqrt{\frac{۱}{۲}} = -۲\frac{۱}{\sqrt{۲}} = -\sqrt{۲}$$

$$a+b=۳$$

$$\left.۲\sqrt{ab}=\sqrt{۵}\right\} \rightarrow ab=\frac{۵}{۴}$$

$$a=\frac{۵}{۲} \quad b=\frac{۱}{۲}$$

$$\frac{\sqrt{۲}}{۲} \times -\sqrt{۲} = -\frac{۲}{۲} = -۱$$

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۳ـ

الف)

$$\frac{۲\sqrt{۲}+۳\sqrt{۳}}{۵-\sqrt{۶}} - ۲\left(\sqrt[۴]{۹}-۱\right)^{-۱} \Rightarrow \frac{۲\sqrt{۲}+۳\sqrt{۳} \times (۵+\sqrt{۶})}{۵-\sqrt{۶} \times (۵+\sqrt{۶})} = \frac{۱۹\sqrt{۲}+۱۹\sqrt{۳}}{\cancel{۲۵-۶}\ ۱۹} = \frac{\cancel{۱۹}(\sqrt{۲}+\sqrt{۳})}{\cancel{۱۹}} = \sqrt{۲}+\sqrt{۳}$$

$$۲\left(\frac{۱}{\sqrt[۴]{۹}-۱}\right) = \frac{۲ \times \sqrt{۳}+۱}{\sqrt{۳}-۱ \times \sqrt{۳}+۱} = \frac{۲\sqrt{۳}+۲}{۳-۱} = \frac{\cancel{۲}(\sqrt{۳}+۱)}{\cancel{۲}} = \sqrt{۳}+۱ \qquad (\sqrt{۲}+\sqrt{۳})-(\sqrt{۳}+۱) = \boxed{\sqrt{۲}-۱}$$

ب)

$$\frac{۳\sqrt{۳}-۱}{۴+\sqrt{۳}} + \frac{۱}{۲-\sqrt{۳}} \qquad \frac{۱ \quad (۲+\sqrt{۳})}{۲-\sqrt{۳} \times (۲+\sqrt{۳})} = \frac{۲+\sqrt{۳}}{۱} \rightarrow ۲+\sqrt{۳}+\sqrt{۳}-۱ = ۲\sqrt{۳}+۱$$

$$\frac{۳\sqrt{۳}-۱ \ (۴-\sqrt{۳})}{۴+\sqrt{۳} \ (۴-\sqrt{۳})} = \frac{۱۲\sqrt{۳}-۹-۴-\sqrt{۳}}{۱۳} = \frac{۱۳\sqrt{۳}-۱۳}{۱۳} = \sqrt{۳}-۱$$

$$\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}-2 \Rightarrow \sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}=A \to A^2=2\sqrt3+2\sqrt{(1+\sqrt3)(\sqrt3-1)}=2\sqrt3+2\sqrt2 \to A=\sqrt{2(\sqrt3+\sqrt2)}$$

$$=\sqrt{\frac{2(\sqrt3+\sqrt2)}{\sqrt3-\sqrt2}}-2 \to \frac{\sqrt3+\sqrt2}{\sqrt3-\sqrt2}\cdot\frac{(\sqrt3+\sqrt2)}{\sqrt3+\sqrt2}=5+2\sqrt6 \to \sqrt{2(5+2\sqrt6)}=\sqrt{10+4\sqrt6}$$

$$(\sqrt6+2)^2=10+4\sqrt6$$

$$\sqrt{10+4\sqrt6}-2=\sqrt6$$

ب) $\sqrt[4]{\sqrt[3]{\sqrt{2^8}}}\times\sqrt[4]{162}\times\sqrt{2^6\sqrt2}=2^{\frac{2}{3}}\times3\times2^{\frac14}\times2^{\frac{13}{4}}=2^{\frac{14}{4}}\times2^{\frac{2}{3}}\times3=2^{\frac{40}{12}}\times3=\underline{2^5\times3}$

$162 = 81\times2$ $\qquad \sqrt{2^6\sqrt2}=\sqrt[4]{\sqrt{2^{13}}}$

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$$\frac{3^x(1+3+9+27+81+243)}{2^{x-2}(1+2+4+8+16+32)}=\frac{3^x\times364}{2^{x-2}\times63}=52 \qquad \text{—۵}$$

$$\frac{52}{9}\times\frac{3^x}{2^{x-2}}=52 \longrightarrow \frac{3^x}{2^{x-2}}=\frac19 \longrightarrow \frac{3^x}{2^{x-2}}\times\frac19=1 \longrightarrow \frac{3^x}{2^{x-2}}=9$$

$$4\left(\frac{3}{2}\right)^x=9 \to \left(\frac32\right)^x=\frac94 \to \boxed{x=2}$$

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$(a^2+b^2+2ab)(a^2+b^2-2ab)$

$= a^4-2a^2b^2+b^4 \longrightarrow \sqrt6-2+\sqrt6+2-2\sqrt2=2\sqrt6-2\sqrt2=t(2-\sqrt3)$ $\qquad (2+\sqrt3)(\sqrt6-\sqrt2)$ $\qquad$ —۶

$$t=\frac{2\sqrt6-2\sqrt2}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3}=6\sqrt2-2\sqrt6+4\sqrt6-4\sqrt2=\boxed{2\sqrt2+2\sqrt6}$$

$$2\left(\underline{\sqrt{\sqrt6-2}\;\sqrt{\sqrt6+2}}\right)$$

$$\sqrt{(\sqrt6-2)(\sqrt6+2)}=2\sqrt{6-4}=2\sqrt2$$

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$$a=\sqrt[4]{(2-\sqrt3)^2}=\sqrt{2-\sqrt3} \qquad a+\frac1a=x \longrightarrow x^4+(\sqrt2)^4-2x^2(\sqrt2)^2 \qquad \text{—۷}$$

$$\left(a+\frac1a\right)^4=a^4+\frac{1}{a^4}+4a^2+\frac{4}{a^2}\to(2-\sqrt3)^2+\cancel{\frac{1}{(2-\sqrt3)^2}}+4(2-\sqrt3)+\frac{4}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3}=8+4\sqrt3\;\Big\}=30$$

$$7-4\sqrt3 \qquad \frac{1}{7+4\sqrt3}\times\frac{7-4\sqrt3}{7-4\sqrt3}=7+4\sqrt3$$

$(a+\frac{1}{a})^{۲} = a^{۲}+\frac{1}{a^{۲}}+۲ \longrightarrow ۲-\cancel{\sqrt{۴}}+۲+\cancel{\sqrt{۴}}+۲ = ۸$

$۳۰+۴-\cancel{\frac{۲\times ۸\times ۲}{۳۲}} = ۳۴-۳۲ = ۲$

$\longrightarrow \boxed{t=1}$

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$\sqrt[۵]{۲^{۲}\sqrt[۳]{۲^{۴}}} = \sqrt[۵]{\sqrt[۳]{۲^{۶}\times ۲^{۴}}} = \sqrt[۱۵]{۲^{۱۰}} = \sqrt[۳]{۲^{۲}} = ۲^{\frac{۲}{۳}}$ , $(\frac{1}{۲})^{-\frac{۴}{۳}} = ۲^{\frac{۴}{۳}}$ / $۲^{\frac{۴}{۳}}\times ۲^{\frac{۲}{۳}} = ۲^{۲} = 4$ -۸

$(۲۸)^{-\frac{1}{۳}} = (۸)^{-\frac{1}{۳}} = \sqrt[۳]{\frac{1}{۸}} = \frac{1}{۲}$

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$(a)^{\frac{1}{۷}} = \underset{۳^{۳}}{۲۷}(a)^{\frac{۱۵}{۷}} \longrightarrow a^{\frac{1}{۷}} = ۳^{۳}\times a^{\frac{۱۵}{۷}} \quad \frac{a^{\frac{۱۵}{۷}}}{a^{\frac{1}{۷}}} = a^{\frac{۱۴}{۷}} = a^{۲} = \frac{1}{۲۷} \longrightarrow a = \frac{1}{۳\sqrt{۳}}$ -۹

$\frac{1}{a}-۳ = ۳\sqrt{۳}-۳ \longrightarrow \frac{۳\sqrt{۳}-۳}{۱+\sqrt{۳}}\times\frac{\sqrt{۳}-1}{\sqrt{۳}-1} = \frac{۹-۳\sqrt{۳}-۳\sqrt{۳}+۳}{۲} = \frac{۱۲-۶\sqrt{۳}}{۲} = ۶-۳\sqrt{۳}$

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$\sqrt{x+a} = p \quad q = \sqrt{x-۴} \quad p-q = ۲ \quad (p-q)(p+q) = p^{۲}-q^{۲}$ -۱۰

$۲(p+q) = (x+a)-(x-۴) = a+۴ \longrightarrow p+q = \frac{a+۴}{۲} \quad p+q-۲ = \frac{a+۴}{۲}-۲ = \frac{a}{۲}$