

پاسخنامه تشریحی تکلیف شماره ۱۱

$$(11 + 31 + 51 + \dots + 71)(21 + 41 + 61 + \dots + 201)$$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $1 + 6 + 5 + \dots + 0 \quad \quad \quad 1 + 6 + 5 + \dots + 0$
 $(7) \times (6) = 42 \rightarrow$

این حاصل
برابر است
با ۲

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الف) $\sqrt[4]{(4+\sqrt{5})^{-1} \times (1+\sqrt{5})^2} = \sqrt[4]{2}$

$\sqrt[4]{\frac{1}{4+\sqrt{5}} \times (1+\sqrt{5})^2} = \sqrt[4]{2}$

ب) $\left(\frac{\sqrt{5}+\sqrt{2}}{\sqrt{2}(\sqrt{5}+\sqrt{2})} \right) \left(\frac{(\sqrt{5}-1)^2}{\sqrt{2}} - \frac{(\sqrt{5}+1)^2}{\sqrt{2}} \right) = \frac{\sqrt{5}-1-\sqrt{5}-1}{2} = -1$

$\sqrt{3}-\sqrt{5} \times 2 \rightarrow 6-2\sqrt{5} = 1+5-2\sqrt{5} \Rightarrow (\sqrt{5}-1)^2 \Rightarrow \frac{(\sqrt{5}-1)^2}{2} \times \sqrt{3}-\sqrt{5} \rightarrow \sqrt{3}+\sqrt{5}$

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الف) $\sqrt[3]{a} = a \rightarrow \frac{a^3+b^3}{a^3+b^3-ab} - 2(b-1)^{-1} = (a+b) - \frac{1}{2} \frac{(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \sqrt{2}+\sqrt{3}-\sqrt{3}-1 = \sqrt{2}-1$

ب) $\sqrt{3}=a \rightarrow \frac{a^3-1}{a^3+1+a} + \frac{1(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{2\sqrt{3}}{2-1} + 2+\sqrt{3} = 2\sqrt{3}+1$

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الف) $\left(\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1} \right)^2 = 1+\sqrt{3}+\sqrt{3}-1+2\sqrt{\sqrt{3}-1} = 2\sqrt{3}+2\sqrt{2} \rightarrow \sqrt{2(\sqrt{3}+\sqrt{2})}$

ب) $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times \frac{1}{6} \times \frac{1}{7} \times \frac{1}{8} \times \frac{1}{9} \times \frac{1}{10} \times \frac{1}{11} \times \frac{1}{12} \times \frac{1}{13} \times \frac{1}{14} \times \frac{1}{15} \times \frac{1}{16} \times \frac{1}{17} \times \frac{1}{18} \times \frac{1}{19} \times \frac{1}{20} = \frac{1}{20!}$

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$\frac{1}{4} (1+3+9+27+81+243) = 52 \rightarrow \left(\frac{w}{y} \right)^{91} = 52 \times \frac{93}{12} \times \frac{1}{364} = \frac{1}{4}$

$\Rightarrow a_1 = 2$

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$$\left((a^r + b^r + \sqrt{ab})(a^r + b^r - \sqrt{ab}) \right)^r = \left((a^r + b^r)^r - (\sqrt{ab})^r \right)^r = \left(a^{4r} + b^{4r} + \cancel{2a^r b^r} - \cancel{2a^r b^r} \right)^r$$

$$\xrightarrow{\text{سؤال}} \left(\sqrt{9} - 2 + \sqrt{9} + 2 - 2\sqrt{(\sqrt{9}+1)(\sqrt{9}-1)} \right)^r = \left(\frac{2\sqrt{9}-2\sqrt{2}}{\sqrt{2}} \right)^r = \frac{2\sqrt{9}-2\sqrt{2}}{\sqrt{2}} = 19(2-\sqrt{2})$$

$$= t(2-\sqrt{2}) \rightsquigarrow \underline{t=19}$$

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$$\left(\left(a + \frac{1}{a} + \sqrt{2} \right) \left(a + \frac{1}{a} - \sqrt{2} \right) \right)^r = \left(a^r + \frac{1}{a^r} + 2 - 2 \right)^r = a^{4r} + \frac{1}{a^{4r}} + 2 = r^2$$

$$\rightarrow a^r = 2 - \sqrt{2} \rightarrow 2 - \sqrt{2} + \frac{1}{2 - \sqrt{2}} + 2 = 19 + 2 = 19 = r^2$$

$$\frac{1(+)}{(-)(+)} = \frac{2 + \sqrt{2}}{4 - 2} = 2 + \sqrt{2} \rightarrow \underline{t=4}$$

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$$A = 2^{\frac{1}{2}} \times 2^{\frac{1}{10}} \times 2^{-1} \times \left(\frac{2}{2} \right)^{\frac{1}{2}} = 2^{\frac{4+2+2-10}{10}} \rightarrow = 2^{\frac{2}{10}} = 2^{\frac{1}{5}}$$

$$\rightarrow (A)^{-\frac{1}{5}} = (2^{\frac{1}{5}})^{-\frac{1}{5}} = (2^{\frac{1}{25}})^{-\frac{1}{5}} = \frac{1}{2} = \underline{0.5}$$

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$$a > 0 \rightarrow \sqrt[3]{a} = 2 \times a^{\frac{10}{3}} \xrightarrow{(\div a^{\frac{10}{3}})} a^{\frac{-1}{3}} = a^{-\frac{1}{3}} = 2^3 \rightarrow \frac{1}{a} = a^{-1} = 2^3 \sqrt{2}$$

$$\frac{1}{a} - 3 = 2^3 \sqrt{2} - 3 = 2(\sqrt{2} - 1)$$

$$\frac{2(\sqrt{2}-1)(\sqrt{2}-1)}{(1+\sqrt{2})(\sqrt{2}-1)} = \frac{2}{2} \times \frac{(\sqrt{2}-1)^2}{2-2} = 2 \times \sqrt{2} \rightarrow \underline{2\sqrt{2}}$$

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$$\left(\sqrt{a+a} + \sqrt{a-a} \right) \left(\sqrt{a+a} - \sqrt{a-a} \right) \xrightarrow{\text{سؤال}} a+a - a+a = a+a \Rightarrow \frac{a+a}{2} = \sqrt{a+a} + \sqrt{a-a}$$

$$\text{خاتمة سؤال: } \sqrt{a+a} + \sqrt{a-a} - 2 = \frac{a+a-a}{2} = \underline{\underline{\frac{a}{2}}}$$

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