(( الینا صیرنیاتی ))

۱۹

۱) $5!$ و بعد به دلیل $2 \times 5 = 10$ همان صفر دارند.

$$\underbrace{(1!+3!)}_{7}\underbrace{(2!+4!)}_{6} = \underline{2}$$

۲ ✓

---

۲) الف) $\sqrt[4]{\frac{1}{4+\sqrt{7}}} \times \sqrt[4]{(1+\sqrt{7})^2} = \sqrt[4]{\frac{1}{9} \times \underbrace{(4-\sqrt{7})(4+\sqrt{7})}_{16-7=9} \times 2} = \sqrt[4]{2}$

ب) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}(\sqrt{5}+\sqrt{2})} = \frac{1}{\sqrt{2}}$ $\quad t^2 = 3-\sqrt{5}+3+\sqrt{5}-4 = 2 \rightarrow |t| = \sqrt{2}$ خود جمله‌ی دو برابر (؟) صفر است

$-\sqrt{2} \times \frac{1}{\sqrt{2}} = \underline{-1}$

۲ ✓

---

۳) الف) $\frac{2\sqrt{2}+3\sqrt{3}\,(5+\sqrt{6})}{5-\sqrt{6}\,(5+\sqrt{6})}$ $\quad \frac{2(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{19\sqrt{2}+19\sqrt{3}}{19} - \sqrt{3}-1$

$\sqrt{2}+\sqrt{3}-\sqrt{3}-1 = \boxed{\sqrt{2}-1}$

۲ ✓

ب) $\frac{\sqrt{27}-1(4-\sqrt{3})}{(4+\sqrt{3})(4-\sqrt{3})} = \frac{13\sqrt{3}-13}{13} = \sqrt{3}-1$

$(2-\sqrt{3})^{-1} = \frac{1}{(2-\sqrt{3})(2+\sqrt{3})} = 2+\sqrt{3}$

$+ \quad \underline{2\sqrt{3}+1}$

---

۴) الف) $\left(\frac{\sqrt{11+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}\right)$ توان ۲ : $t$ صفر $= \frac{12+\sqrt{3}-11+\sqrt{3}+2\sqrt{2}(\sqrt{3}-\sqrt{2})}{\sqrt{3}-\sqrt{2} \times (\sqrt{3}-\sqrt{2})} \xrightarrow{\text{جذر}} \sqrt{6}+2$

$\rightarrow t-2 = \sqrt{6}$ ✓

۱

۴ → ب → $\sqrt[4]{\sqrt[3]{2^8}} \times \sqrt[4]{162} \times \sqrt{4\sqrt[4]{2}}$

$(2^8)^{\frac{1}{12}} \times 3 \times \sqrt[4]{2} \times 2 \times (2)^{\frac{1}{8}} = 2^{\frac{2}{3}} \times 3 \times 2^{\frac{1}{4}} \times 2^{\frac{13}{8}} = 3 \times 2^2 = 12$

(۴) $\left[(a^2+b^2)^2-(2ab)^2\right]^2=\left[(a^4+b^4+2a^2b^2)-4a^2b^2\right]^2$

$=\left[a^4+b^4-2a^2b^2\right]^2=\sqrt{6}-2+\sqrt{6}+2-2\left(\sqrt[4]{\sqrt{6}+2}\right)\left(\sqrt[4]{\sqrt{6}-2}\right)$

$2\sqrt{6}-2\sqrt{2}=$ $\cancel{\sqrt{6-4}}$ $\sqrt{2}$

$\longrightarrow \left[2(\sqrt{6}-\sqrt{2})\right]^2=4(8-2\times2\sqrt{12})$

$16(2-\sqrt{3})\longrightarrow \boxed{t=16}$

۲

(۵) $\left[\left(a+\frac{1}{a}\right)^2-2\right]^2=\left[\left(a^2+\frac{1}{a^2}+2-2\right)\right]^2=\left(\frac{1}{a^2}+a^2\right)^2$

$\frac{1}{a^4}+a^4+2=7-4\sqrt{3}+\frac{1}{7-4\sqrt{3}}+2=7-4\sqrt{3}+\frac{7+4\sqrt{3}}{49-48}+2$

$14+2=2^t$ $16=2^t\longrightarrow \boxed{t=4}$

$4\times14=$
$56$

۲

۵

$$\frac{۳^x\,(\cancel{۱+۳+\dots+۲۴۳})^{۳۶۴}}{۲^x\,(\cancel{\frac{۱}{۴}+\frac{۱}{۲}+\dots+۸})_{\frac{۶۳}{۴}}} = ۵۲ \Longrightarrow ۲ \text{ :گزینه}$$

$$\frac{۳^x \times ۷}{۲^x \times \frac{۶۳}{۴}} = ۱$$

$$\frac{۴ \times ۳^x}{۲^x \times ۹} = ۱ \longrightarrow \left(\frac{۳}{۲}\right)^x = \frac{۹}{۴} \Longrightarrow \boxed{x=۲}$$

۲ ✓

۴

(۸) $2^{\frac{2}{5}} \times 2^{\frac{4}{15}} \times 2^{\frac{4}{3}} = 2^{\frac{6+4+20}{15}} = 2^{\frac{30}{15}} = 2^{2} = 4$

$(2\times 4)^{-\frac{1}{3}} = \left(\frac{1}{8}\right)^{\frac{1}{3}} = \frac{1}{(2^{3})^{\frac{1}{3}}} = \frac{1}{2} = \underbrace{2^{-1}}_{3}$ ✓

---

(۹) $\sqrt[4]{a}$ … $a^{\frac{15}{7}}$ … $-\frac{14}{\,}$ … $3$ … $-2$ … $3$

$$\to \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{1-3}$$

$$-\sqrt{3}\;1$$

$$=$$

(۹) $a^{\frac{1}{7}} = 3^{3}\, a^{\frac{15}{7}} \to a^{-2} = 3^{3} \to \frac{1}{a} = 3\sqrt{3}$

$$\frac{3(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3(3+1-2\sqrt{3})}{3-1} = \frac{9+3-6\sqrt{3}}{2}$$

$$= \frac{12-6\sqrt{3}}{2} = \boxed{6-3\sqrt{3}}$$

۲ ✓

(۱۰) $\sqrt{x+a} - \sqrt{x-4} = 2 \xrightarrow{\times\, \sqrt{x+a}+\sqrt{x-4}}$ $\quad x+a-x+4 = 2(\sqrt{x+a}+\sqrt{x-4})$

$$\frac{a}{2}+2 = \sqrt{x+a}+\sqrt{x-4} \to \frac{a}{2} = \sqrt{x+a}+\sqrt{x-4}-2$$

۲ ✓