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آترین مجلسی   پاسخ تکلیف ۲۱   کلاس ج

۱۹,۷۵

①

$$(1!+3!+\underbrace{5!+\dots+25!}_{\text{یکان صفر}})(2!+4!+\underbrace{6!+\dots+26!}_{\text{یکان صفر}})$$

$1+6=7$ $2+4=6$

$$\downarrow$$

$$7\times 6=42 \quad (2)$$

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② الف)

$$\sqrt[4]{(4+\sqrt7)^{-1}} \times \sqrt{1+\sqrt7} = \sqrt[4]{(4+\sqrt7)^{-1}} \times \sqrt[2\times2]{(1+\sqrt7)^2}$$

$$= \sqrt[4]{(4+\sqrt7)^{-1}} \times \sqrt[4]{1+7+2\sqrt7} = \sqrt[4]{(4+\sqrt7)^{-1}(8+2\sqrt7)} = \sqrt[4]{\frac{1}{4+\sqrt7}\times 2(4+\sqrt7)} = \sqrt[4]{2}$$

ب) $A=\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}$

$$A^2=(3-\sqrt5)+(3+\sqrt5)-2\sqrt{3-\sqrt5}\times\sqrt{3+\sqrt5} = 6-2\sqrt{(3-\sqrt5)(3+\sqrt5)}$$

$$=6-2\sqrt4=6-4=2 \longrightarrow A^2=2 \xrightarrow{A<0} A=-\sqrt2$$

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$$\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)(-\sqrt2) = \frac{-(\sqrt4+\sqrt{10})}{\sqrt{10}+2} = \frac{-(2+\sqrt{10})}{\sqrt{10}+2} = -1$$

③ الف)

$$A=\frac{\sqrt8+\sqrt{27}}{5-\sqrt6} = \frac{2\sqrt2+3\sqrt3}{5-\sqrt6}\times\frac{5+\sqrt6}{5+\sqrt6} = \frac{10\sqrt2+15\sqrt3+(2\sqrt2)(\sqrt6)+(3\sqrt3)(\sqrt6)}{5^2-6}$$

$$\begin{cases}(2\sqrt2)(\sqrt6)=2\sqrt2(\sqrt2\times\sqrt3)=4\sqrt3\\(3\sqrt3)(\sqrt6)=3\sqrt3(\sqrt3\times\sqrt2)=9\sqrt2\end{cases}$$

$$A=\frac{10\sqrt2+15\sqrt3+4\sqrt3+9\sqrt2}{19}=\frac{19\sqrt2+19\sqrt3}{19}=\sqrt2+\sqrt3$$

$$\sqrt[4]9=\sqrt[4]{3^2} \longrightarrow B=2(\sqrt[4]9-1)^{-1}=\frac{2}{\sqrt[4]9-1}=\frac{2}{\sqrt3-1}\times\frac{\sqrt3+1}{\sqrt3+1}=\frac{2(\sqrt3+1)}{\underbrace{3-1^2}_{2}}=\sqrt3+1$$

$$A-B=\sqrt2+\sqrt3-(\sqrt3+1)=\boxed{\sqrt2-1}$$

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ب)

$$A=\frac{\sqrt{27}-1}{4+\sqrt3}=\frac{3\sqrt3-1}{4+\sqrt3}\times\frac{4-\sqrt3}{4-\sqrt3}=\frac{12\sqrt3-9-4+\sqrt3}{4^2-3}=\frac{13\sqrt3-13}{13}=\sqrt3-1$$

$$B=(2-\sqrt3)^{-1}=\frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3}=\frac{2+\sqrt3}{2^2-3}=2+\sqrt3$$

$$A+B=(\sqrt3-1)+(2+\sqrt3)=\boxed{1+2\sqrt3}$$

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الف) $A=\sqrt{1+\sqrt{۳}}+\sqrt{\sqrt{۳}-1} \longrightarrow A^{۲}=(1+\sqrt{۳})+(\sqrt{۳}-1)+۲\sqrt{(\sqrt{۳}+1)(\sqrt{۳}-1)}$ ④

$A^{۲}=۲\sqrt{۳}+۲\sqrt{۲}=۲(\sqrt{۳}+\sqrt{۲})$

$A=\sqrt{۲(\sqrt{۳}+\sqrt{۲})}$

$\frac{\sqrt{1+\sqrt{۳}}+\sqrt{\sqrt{۳}-1}}{\sqrt{\sqrt{۳}-\sqrt{۲}}}=\frac{\sqrt{۲(\sqrt{۳}+\sqrt{۲})}}{\sqrt{\sqrt{۳}-\sqrt{۲}}}\times\frac{\sqrt{\sqrt{۳}+\sqrt{۲}}}{\sqrt{\sqrt{۳}+\sqrt{۲}}}=\frac{\sqrt{۲(\sqrt{۳}+\sqrt{۲})^{۲}}}{\sqrt{۳}-۲}=\sqrt{۲}\times(\sqrt{۳}+\sqrt{۲})=\sqrt{۶}+۲$

$(\sqrt{۶}+۲)-۲=\boxed{\sqrt{۶}}$

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ب) $\sqrt[۴]{۲^{\frac{۵}{۴}}}\times\sqrt[۴]{۲\times۳^{۴}}\times\sqrt{۲^{۲}\sqrt[۴]{۲}}=۲^{\frac{۵}{۱۶}}\times۳\sqrt[۴]{۲}\times\sqrt{\sqrt[۶]{۲^{۱۲}\times۲}}=\sqrt[۳]{۲^{۳}}\times۳\sqrt[۳]{۲}\times۲\sqrt[۱۲]{۲}$

$=\sqrt[۱۲]{۲^{۹}}\times۳\sqrt[۱۲]{۲^{۳}}\times۲\sqrt[۳]{۲}=\sqrt[۱۲]{۲^{۱۲}}=\boxed{۲}\times۳\times۲=۱۲$

$\frac{۳^{x}+۳^{x+1}+۳^{x+۲}+۳^{x+۳}+۳^{x+۴}+۳^{x+۵}}{۲^{x-۲}+۲^{x-1}+۲^{x}+۲^{x+1}+۲^{x+۲}+۲^{x+۳}}=۵۲$ ⑤

$\frac{۳^{x}(1+۳+۳^{۲}+۳^{۳}+۳^{۴}+۳^{۵})}{۲^{x-۲}(1+۲+۲^{۲}+۲^{۳}+۲^{۴}+۲^{۵})}=۵۲\longrightarrow\frac{۳^{x}(1+۳+۹+۲۷+۸1+۲۴۳)}{۲^{x-۲}(1+۲+۴+۸+1۶+۳۲)}=۵۲$

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$\frac{۳^{x}\times۳۶۴}{۲^{x-۲}\times۶۳}=۵۲\longrightarrow\frac{۳^{x}}{۲^{x-۲}}=\frac{۶۳\times۵۲}{۳۶۴}\longrightarrow\frac{۳^{x}}{۲^{x-۲}}=۹\longrightarrow\frac{۳^{x}}{۲^{x}\times۲^{-۲}}=۹\longrightarrow\frac{۳^{x}}{۲^{x}}=\frac{۹}{۴}$

$(\frac{۳}{۲})^{x}=(\frac{۳}{۲})^{۲}\longrightarrow x=۲$

جواب ۶ در صفحه بعد

الف) $(a+\frac{1}{a}+\sqrt{۲})^{۲}(a+\frac{1}{a}-\sqrt{۲})^{۲}=((a+\frac{1}{a})^{۲}-(\sqrt{۲})^{۲})^{۲}$ ⑦

$=(a^{۲}+\frac{1}{a^{۲}}+۲a(\frac{1}{a})-۲)^{۲}=(a^{۲}+\frac{1}{a^{۲}})^{۲}$ $۷-۴\sqrt{۳}=(۲-\sqrt{۳})^{۲}$

$a=\sqrt[۴]{۷-۴\sqrt{۳}}=\sqrt[۴]{(۲-\sqrt{۳})^{۲}}=\sqrt{۲-\sqrt{۳}}$ $(a^{۲}+\frac{1}{a^{۲}})^{۲}=(۲-\sqrt{۳}+\frac{1}{۲-\sqrt{۳}})^{۲}$

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$\frac{1}{۲-\sqrt{۳}}=۲+\sqrt{۳}$ $(۲-\sqrt{۳}+۲+\sqrt{۳})^{۲}=۴^{۲}\longrightarrow۲^{t}=۴^{۲}=۲^{۴}\longrightarrow t=۴$

$$A = (a^2+b^2-2ab)^2 (a^2+b^2+2ab)^2$$ (4)

$$= ((a-b)^2)^2 ((a+b)^2)^2 = ((a-b)^2(a+b)^2)^2 = ((a^2-b^2)^2)^2 = (a^4+b^4-2a^2b^2)^2$$

$$\begin{cases} a = \sqrt[4]{\sqrt{6}-2} \longrightarrow a^4 = \sqrt{6}-2 \,,\; a^2 = \sqrt{\sqrt{6}-2} \\ b = \sqrt[4]{\sqrt{6}+2} \longrightarrow b^4 = \sqrt{6}+2 \,,\; b^2 = \sqrt{\sqrt{6}+2} \end{cases}$$

$$2a^2b^2 = 2\sqrt{\sqrt{6}-2} \times \sqrt{\sqrt{6}+2} = 2\sqrt{(\sqrt{6}-2)(\sqrt{6}+2)} = 2\sqrt{6-4} = 2\sqrt{2}$$

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$$A = ((\sqrt{6}-2)+(\sqrt{6}+2)-2\sqrt{2})^2 = (2\sqrt{6}-2\sqrt{2})^2 = 4(\sqrt{6}-\sqrt{2})^2$$

$$= 4(6+2-2\sqrt{12}) = 4(8-4\sqrt{3}) = 16(2-\sqrt{3})$$

$$\longrightarrow t = 16$$

$$A = \sqrt[5]{4\sqrt[3]{16}}\left(\frac{1}{2}\right)^{-\frac{4}{3}} = \sqrt[5]{\sqrt[3]{4^3 \times 16}}\,(2^{-1})^{-\frac{4}{3}} = \sqrt[5\times 3]{4^3 \times 4^2} \times 2^{\frac{4}{3}} =$$ (5)

$$= \sqrt[5\times 3]{4^5} \times 2^{\frac{4}{3}} = \sqrt[3]{4} \times 2^{\frac{4}{3}} = 4^{\frac{1}{3}} \times 2^{\frac{4}{3}} = (2^2)^{\frac{1}{3}} \times 2^{\frac{4}{3}} = 2^{\frac{6}{3}} = 2^2$$

$$(2^A)^{-\frac{1}{3}} = (2 \times 2^2)^{-\frac{1}{3}} = (2^3)^{-\frac{1}{3}} = 2^{-1} = \frac{1}{2} = 0/5$$ 2

$$\sqrt[7]{a} = 27 \times a^{\frac{15}{7}} \longrightarrow a^{\frac{1}{7}} = 27 \times a^{\frac{15}{7}} \longrightarrow \frac{a^{\frac{1}{7}}}{a^{\frac{15}{7}}} = 27$$ (9)

$$a^{\frac{1}{7}-\frac{15}{7}} = 27 \longrightarrow a^{-\frac{14}{7}} = 27 \rightarrow a^{-2} = 27 \longrightarrow \sqrt{a^{-2}} = \sqrt{27} \longrightarrow a^{-1} = 3\sqrt{3} \longrightarrow \frac{1}{a} = 3\sqrt{3}$$

$$\frac{\frac{1}{a}-3}{1+\sqrt{3}} = \frac{3\sqrt{3}-3}{\sqrt{3}+1} = \frac{3(\sqrt{3}-1)}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3(\sqrt{3}-1)^2}{3-1} = \frac{3(3+1-2\sqrt{3})}{2}$$

$$= \frac{3(4-2\sqrt{3})}{2} = 3(2-\sqrt{3}) = 6-3\sqrt{3}$$

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$(\sqrt{x+a} - \sqrt{x-۴})(\sqrt{x+a} + \sqrt{x-۴}) \rightarrow ۲(\sqrt{x+a} + \sqrt{x-۴})$

$\rightarrow (\sqrt{x+a})^۲ - (\sqrt{x-۴})^۲ \rightarrow ۲(\sqrt{x+a} + \sqrt{x-۴}) \rightarrow (x+a) - (x-۴)$

$\rightarrow ۲(\sqrt{x+a} + \sqrt{x-۴}) \longrightarrow a+۴ = ۲(\sqrt{x+a} + \sqrt{x-۴}) \longrightarrow \sqrt{x+a} + \sqrt{x-۴} = \frac{a+۴}{۲}$

$\longrightarrow \sqrt{x+a} + \sqrt{x-۴} = \frac{a}{۲} + ۲ \longrightarrow \boxed{\sqrt{x+a} + \sqrt{x-۴} - ۲ = \frac{a}{۲}}$

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PAPCO