

$$\left( \frac{11}{1} + \frac{31}{6} \right) \left( \frac{21}{2} + \frac{41}{12} \right) \leq 42 \rightarrow 2$$

✓

الف)  $\sqrt{(4+\sqrt{5})^{-1} \times \frac{1+\sqrt{5}}{4+\sqrt{5}}} = \sqrt{\frac{2(4+\sqrt{5})}{(4+\sqrt{5})}} = \sqrt{2}$

ب)  $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}\sqrt{5}+\sqrt{2}} = \frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}(\sqrt{2}\sqrt{5}+1)} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \frac{\sqrt{2}}{2}(\sqrt{2}-\sqrt{5}-\sqrt{3}+\sqrt{5}) = \frac{1}{2}(\sqrt{2}-\sqrt{3}-\sqrt{5}+\sqrt{5}) = \frac{1}{2}(\sqrt{2}-\sqrt{3}-1+\sqrt{5}) \leq -1$

✓

الف)  $\frac{\sqrt{8}+\sqrt{2}\sqrt{5}}{5-\sqrt{5}} \times \frac{5+\sqrt{5}}{5+\sqrt{5}} = \frac{4\sqrt{8}+5\sqrt{2}\sqrt{5}+\sqrt{48}+\sqrt{10}}{25-5} = \frac{12\sqrt{2}+10\sqrt{5}+4\sqrt{3}+4\sqrt{2}}{20} = \frac{3\sqrt{2}+5\sqrt{5}+\sqrt{3}+\sqrt{2}}{5}$

ب)  $\frac{2}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{2(\sqrt{2}+1)}{2} = \sqrt{2}+1$

ج)  $\frac{3\sqrt{2}-1}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}} = \frac{15\sqrt{2}-5-9+\sqrt{2}}{25-2} = \frac{16\sqrt{2}-14}{23} = \frac{16\sqrt{2}-14}{23}$

د)  $\frac{1}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{2+\sqrt{2}}{4-2} = \frac{2+\sqrt{2}}{2} = 1+\frac{\sqrt{2}}{2}$

✓

الف)  $(\sqrt{3}+1) + \sqrt{3}-1 = 2\sqrt{3}$

ب)  $\sqrt{2(\sqrt{2}+\sqrt{2})} = \sqrt{2(2\sqrt{2})} = \sqrt{4\sqrt{2}} = 2\sqrt[4]{2}$

ج)  $2^{\frac{1}{12}} \times 2^{\frac{1}{12}} \times 2^{\frac{1}{12}} \times 2^{\frac{1}{12}} \times 2^{\frac{1}{12}} \times 2^{\frac{1}{12}} \times 2^{\frac{1}{12}} \times 2^{\frac{1}{12}} = 2^{\frac{8}{12}} = 2^{\frac{2}{3}} = \sqrt[3]{4}$

✓

الف)  $2^m (1 + 3 + 9 + 27 + 81 + 243) \leq 2^m \times \frac{1}{512} \Rightarrow \frac{2^m \times 342}{2^m \times \frac{1}{512}} \leq 1$

ب)  $2^m \left( \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \frac{1}{256} \right) \leq 1$

✓

ج)  $\frac{2^m \times \frac{1}{256}}{2^m \times \frac{1}{4}} \leq 1 \Rightarrow \left( \frac{2}{5} \right)^m = \frac{4}{5} = \left( \frac{2}{5} \right)^2 \rightarrow m = 2$

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| $((a-b)^r)^r ((a+b)^r)^r = ((a-b)(a+b))^r = ((a^r-b^r)^r)^r = (a^r - r a^r b^r + b^r)^r$ $a^r = \sqrt{4-r} \quad b^r = \sqrt{4+r} \quad a^r b^r = (\sqrt{4-r})^r \times (\sqrt{4+r})^r = \sqrt{4-r} \times \sqrt{4+r} = \sqrt{(4-r)(4+r)} = \sqrt{4-r^2} = \sqrt{4}$ $\Rightarrow (\sqrt{4-r} - r\sqrt{4+r} + \sqrt{4+r})^r = (r\sqrt{4} - r\sqrt{r})^r = r(\sqrt{4}-\sqrt{r})^r = r(4+r-r\sqrt{r})^r$ $r(1-r\sqrt{r}), 14(r-\sqrt{r}) = 2(r-\sqrt{r}) \Rightarrow \boxed{r=14} \quad \textcircled{v} \quad \checkmark$                                        | <p>8</p>  |
| $a = \frac{r}{\sqrt{r-\sqrt{r}}} \quad a^r = r - \sqrt{r} \quad (a + \frac{1}{a} + \sqrt{r})^r (a + \frac{1}{a} - \sqrt{r})^r =$ $\frac{1}{a^r} = \frac{1}{r-\sqrt{r}} \times \frac{r+\sqrt{r}}{r+\sqrt{r}} = \frac{r+\sqrt{r}}{r-\sqrt{r}} \times r + \sqrt{r} \left( (a + \frac{1}{a})^r - (\sqrt{r})^r \right)^r = (a^r + \frac{1}{a^r} + \sqrt{r})^r$ $(a^r + \frac{1}{a^r})^r = (r - \sqrt{r} + r + \sqrt{r})^r = 14 \times r^r \Rightarrow \boxed{r=14} \quad \textcircled{v} \quad \checkmark$                                                          | <p>7</p>  |
| $A = \sqrt[r]{r^r} \times \sqrt[r]{r^r} \times r^{\frac{r}{r}} = \sqrt[r]{r^r \times r^r} \times r^{\frac{r}{r}} = \sqrt[r]{r^{\frac{1}{r} \times \frac{r}{r}} \times r^{\frac{r}{r}}} \times r^{\frac{r}{r}} = r \times r^{\frac{r}{r}} = r$ $r(A)^{-\frac{1}{r}} = \Lambda^{-\frac{1}{r}} = r^{\frac{r}{r} \times -\frac{1}{r}} = \boxed{\frac{1}{r}} \quad \textcircled{v} \quad \checkmark$                                                                                                                                                                | <p>1</p>  |
| $\sqrt[r]{a} = r \sqrt[r]{a^{\frac{1}{r}}} \rightarrow a = r \sqrt[r]{a^{\frac{1}{r}}} \quad a^{\frac{1}{r}} = r^{-\frac{1}{r}} \quad a = r^{-\frac{1}{r} \times \frac{r}{r}} = \frac{1}{\sqrt[r]{r}} = \frac{1}{r^{\frac{1}{r}}}$ $\frac{1}{a} = r = r \sqrt[r]{r} = r = r(\sqrt[r]{r}-1) \quad \frac{r(\sqrt[r]{r}-1)(\sqrt[r]{r}-1)}{(\sqrt[r]{r}+1)(\sqrt[r]{r}-1)} = \frac{r(\sqrt[r]{r}-1)^r}{r} = \frac{1}{\sqrt[r]{r}} = \frac{1}{r^{\frac{1}{r}}}$ $\frac{r}{r} \sqrt[r]{r+1-r\sqrt{r}} = \boxed{4-3\sqrt{r}} \quad \textcircled{v} \quad \checkmark$ | <p>9</p>  |
| $\sqrt{n+a} - \sqrt{n-r} = r \quad A-B=r \quad A^r-B^r = x+a-n+r = a+r$ $\frac{(A-B)(A+B)}{r} = a+r \quad A+B = \frac{a+r}{r}$ $\sqrt{n+a} + \sqrt{n-r} - r = A+B-r = \frac{a+r}{r} - \frac{r}{r} = \boxed{\frac{a}{r}} \quad \textcircled{v} \quad \checkmark$                                                                                                                                                                                                                                                                                                | <p>10</p> |