نام و نام خانوادگی ...... آیدا علی عسگری ...... پاسخنامه تشریحی تکلیف شماره ۲۱... کلاس ......

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$$\left(\underbrace{\frac{11}{1}}_{1} + \underbrace{\frac{41}{6}}_{6}\right)\left(\frac{21}{2} + \frac{41}{24}\right) = 42 \to \boxed{2}$$

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الف) $\sqrt[4]{(4+\sqrt{7})^{-1} \times \frac{(1+\sqrt{7})^2}{\cancel{8+2\sqrt{7}}\ 2(4+\sqrt{7})}} = \sqrt[4]{\frac{2\cancel{(4+\sqrt{7})}}{\cancel{(4+\sqrt{7})}}} = \boxed{\sqrt[4]{2}}$

ب) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}\sqrt{5}+(\sqrt{2})^2} = \frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}(\sqrt{2}+\sqrt{5})} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \frac{\sqrt{2}}{2}\left(\sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}}\right) = \frac{1}{2}\left(\sqrt{6-2\sqrt{5}} - \sqrt{6+2\sqrt{5}}\right)$

$\sqrt{(1-\sqrt{5})^2}$ $\sqrt{(1+\sqrt{5})^2}$

$\frac{1}{2}\left(\cancel{\sqrt{5}}-1-1-\cancel{\sqrt{5}}\right) = \boxed{-1}$

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الف) $\frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{6}} \times \frac{5+\sqrt{6}}{5+\sqrt{6}} = \frac{5\sqrt{8}+5\sqrt{27}+\sqrt{48}+\sqrt{162}}{25-6} = \frac{10\sqrt{2}+15\sqrt{3}+4\sqrt{3}+9\sqrt{2}}{19} = \sqrt{2}+\sqrt{3}$

$\frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{2(\sqrt{3}+1)}{2} = \sqrt{3}+1$ $\quad \sqrt{3}+\sqrt{2}-\sqrt{3}-1 = \boxed{\sqrt{2}-1}$

ب) $\frac{3\sqrt{3}-1}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{12\sqrt{3}-4-9+\sqrt{3}}{16-3} = \frac{13\sqrt{3}-13}{13} = \sqrt{3}-1$

$\frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2+\sqrt{3}$ $\quad \sqrt{3}-1+2+\sqrt{3} = \boxed{2\sqrt{3}+1}$

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الف) $\left(\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1}\right)^2 = 1+\sqrt{3}+\sqrt{3}-1+2\sqrt{(1+\sqrt{3})(\sqrt{3}-1)} = 2\sqrt{3}+2\sqrt{2} = 2(\sqrt{3}+\sqrt{2})$

$\Rightarrow$ صورت $\sqrt{2(\sqrt{3}+\sqrt{2})} \Rightarrow \sqrt{\frac{2(\sqrt{3}+\sqrt{2})\times\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}\times\sqrt{3}+\sqrt{2}}} = \sqrt{\frac{2(\sqrt{3}+\sqrt{2})^2}{\cancel{3-2}}} = \sqrt{2(5+2\sqrt{6})} = \sqrt{10+4\sqrt{6}} = \sqrt{6}+2$

$(\sqrt{6}+2)-2 = \boxed{\sqrt{6}}$

ب) $2^{\frac{8}{12}} \times 2^{\frac{3}{12}\cancel{\frac{1}{4}}} \times 2^{\frac{13}{12}} \times 3^{\cancel{4}} = 2^{\frac{24}{12}} \times 3 = 4 \times 3 = \boxed{12}$

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$$\left(\frac{3^n\left(1+3+9+\cancel{27+81+243}\right)^{364}}{2^n\left(\frac{1}{4}+\cancel{\frac{1}{2}+1+2}+4+8\right)_{\frac{63}{4}}} = 52\right) \times \frac{1}{52} \Rightarrow \frac{3^n \times 7}{2^n \times \frac{63}{4}} = 1$$

$$\frac{3^n \times \cancel{28}^{4}}{2^n \times \cancel{63}_{9}} = 1 \Rightarrow \left(\frac{3}{2}\right)^n = \frac{9}{4} = \left(\frac{3}{2}\right)^2 \to \boxed{n=2}$$

$((a-b)^2)^2\,((a+b)^2)^2 = ((a-b)(a+b))^2)^2 = ((a^2-b^2)^2)^2 = (a^4 - 2a^2b^2 + b^4)^2$ $a^4 = \sqrt{6}-2 \quad b^4 = \sqrt{6}+2 \quad a^2b^2 = (\sqrt[4]{\sqrt{6}-2})^2 \times (\sqrt[4]{\sqrt{6}+2})^2 = \sqrt{\sqrt{6}-2} \times \sqrt{\sqrt{6}+2} =$ $\sqrt{(\sqrt{6}-2)(\sqrt{6}+2)} = \sqrt{6-4} = \sqrt{2}$ $\Rightarrow (\sqrt{6}-2-2\sqrt{2}+\sqrt{6}+2)^2 = (2\sqrt{6}-2\sqrt{2})^2 = 4(\sqrt{6}-\sqrt{2})^2 = 4(6+2-\cancel{2\sqrt{12}}^{4\sqrt{3}})$ $4(8-4\sqrt{3}) = 16(2-\sqrt{3}) = t(2-\sqrt{3}) \Rightarrow \boxed{t=16}$	۶
$a = \sqrt[\cancel{4}\,2]{(2-\sqrt{3})^{\cancel{2}}} \quad a^2 = 2-\sqrt{3} \qquad (a+\frac{1}{a}+\sqrt{2})^2(a+\frac{1}{a}-\sqrt{2})^2 =$ $\frac{1}{a^2} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{\cancel{4-3}_{1}} = 2+\sqrt{3} \quad ((a+\frac{1}{a})^2 - (\sqrt{2})^2)^2 = (a^2+\frac{1}{a^2}+\cancel{2-2})^2$ $(a^2+\frac{1}{a^2})^2 = (2-\sqrt{3}+2+\sqrt{3})^2 = 16 = 2^t \Rightarrow \boxed{t=4}$	۷
$A = \sqrt[5]{2^2 \times \sqrt[3]{2^4}} \times 2^{\frac{4}{3}} = \sqrt[5]{2^2 \times 2^{\frac{4}{3}}} \times 2^{\frac{4}{3}} = \sqrt[5]{2^{\frac{10}{3}}} \times 2^{\frac{4}{3}} = 2^{\frac{2}{3}+\frac{4}{3}} = 4$ $2(A)^{-\frac{1}{3}} = 8^{-\frac{1}{3}} = 2^{3 \times -\frac{1}{3}} = \boxed{\frac{1}{2}}$	۸
$\sqrt[7]{a} = 27\,a^{\frac{15}{7}} \longrightarrow a = 27^7 a^{15} \quad a^{14} = 3^{-21} \quad a = 3^{\frac{-21}{14}} = 3^{\frac{-3}{2}} = \frac{1}{\sqrt{27}} = \frac{1}{3\sqrt{3}}$ $\frac{1}{a} - 3 = 3\sqrt{3}-3 = 3(\sqrt{3}-1) \quad \frac{3(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3(\sqrt{3}-1)^2}{2}$ $\frac{3}{\cancel{2}} \times (\cancel{3+1-2\sqrt{3}}) = \boxed{6-3\sqrt{3}}$	۹
$\sqrt{x+a} - \sqrt{x-4} = 2 \quad A-B = 2 \quad A^2-B^2 = x+a-x+4 = a+4$ $\underbrace{\cancel{(A-B)}}_{2}(A+B) = a+4 \quad A+B = \frac{a+4}{2}$ $\underbrace{\sqrt{x+a}+\sqrt{x-4}}_{A+B} - 2 = A+B-2 = \frac{a+4}{2} - \frac{4}{2} = \boxed{\frac{a}{2}}$	۱۰