

المسألة ١٨١٥

$$(1! + 2! + 3! + \dots + n!) / (1! + 2! + 3! + \dots + (n-1)!) = \frac{1}{n}$$

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$n \times 4 = 4 \times n$$

✓

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$$\left(\frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}} \right) \left(\sqrt{2} - \sqrt{3} - \sqrt{2} + \sqrt{3} \right)$$

$$\frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}} = \frac{1}{\sqrt{2} + \sqrt{3}}$$

$$\frac{1}{\sqrt{2}} \times \sqrt{2} = 1$$

✓

$$\frac{\sqrt{2} - 1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = \frac{(\sqrt{2} - 1)^2}{2 - 1} = \sqrt{2} - 1$$

$$\frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} = \frac{\sqrt{2} + 1}{1}$$

$$\sqrt{2} - 1 + \sqrt{2} + 1 = 2\sqrt{2}$$

✓

$$\frac{1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = \frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1$$

$$\frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} = \frac{\sqrt{2} + 1}{1}$$

$$\Rightarrow \sqrt{2} - 1 + \sqrt{2} + 1 = 2\sqrt{2}$$

✓

$$\frac{1}{\sqrt{2} + \sqrt{3}} \times \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}} = \frac{\sqrt{2} - \sqrt{3}}{2 - 3} = \sqrt{3} - \sqrt{2}$$

$$\frac{1}{\sqrt{2} + \sqrt{3}} \times \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}} = \frac{\sqrt{2} - \sqrt{3}}{-1} = \sqrt{3} - \sqrt{2}$$

$$\frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}} \times \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}} = \frac{(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})}{2 - 3} = \frac{2 - 3}{-1} = 1$$

$$\frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} = \frac{\sqrt{2} + 1}{1}$$

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$$\sqrt{2} - 1$$

$$\frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}} \times \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}} = \frac{(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})}{2 - 3} = \frac{2 - 3}{-1} = 1$$

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$$\left(\frac{(\sqrt{4+x} + \sqrt{4-x}) - 2\sqrt{2}}{(\sqrt{4+x} + \sqrt{4-x}) + 2\sqrt{2}} \right)^2 \Rightarrow$$

$$(\sqrt{4+x} + \sqrt{4-x})^2 - 4\sqrt{2} \Rightarrow \sqrt{4+x} + \sqrt{4-x} + 2\sqrt{2} - 4\sqrt{2} = (\sqrt{4+x} - \sqrt{4-x})^2$$

~~$$\Rightarrow \sqrt{4+x} - \sqrt{4-x} = 2\sqrt{2}$$~~

$$2\sqrt{2} + 1 = \sqrt{4+x} - \sqrt{4-x}$$

$$14 - 14\sqrt{2} + 14(2 - \sqrt{2}) = 4 + 14$$

$$A = r^{\frac{1}{2}} \times r^{\frac{1}{2}} \times r^{\frac{1}{2}} \times r^{\frac{1}{2}} \times r^{\frac{1}{2}} \times r^{\frac{1}{2}} \times r^{\frac{1}{2}} \times r^{\frac{1}{2}}$$

$$(r_2 r^2)^{-\frac{1}{2}} (r^2)^{-\frac{1}{2}} =$$

$$r^{-1} \times \frac{1}{r} = \frac{1}{r^2}$$

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$$\sqrt{a} = a^{\frac{1}{2}} \Rightarrow a^{\frac{1}{2}} = \sqrt{a} \Rightarrow a^{-\frac{1}{2}} = \frac{1}{\sqrt{a}}$$

$$\frac{2\sqrt{3}-1}{1+\sqrt{3}} \times \frac{2\sqrt{3}-1}{\sqrt{3}-1} = \frac{2\sqrt{3}-1}{\sqrt{3}-1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$\frac{2(1+3-2\sqrt{3})}{3-1} = \frac{2(4-2\sqrt{3})}{2} = 2(2-\sqrt{3}) = 4-2\sqrt{3}$$

$$a = \sqrt{(2-\sqrt{2})^2} = \sqrt{2-\sqrt{2}}$$

$$\left(a + \frac{1}{a}\right)^2 - 2 = \left(a^2 + \frac{1}{a^2} + 2 - 2\right)^2 = \left(a^2 + \frac{1}{a^2}\right)^2 \Rightarrow$$

~~$$\frac{1}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{2+\sqrt{2}}{2-\sqrt{2}}$$~~

$$2-\sqrt{2} + \frac{1}{2-\sqrt{2}} = 2-\sqrt{2} + 2+\sqrt{2} = 4$$

$$\frac{1}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{2+\sqrt{2}}{2-\sqrt{2}}$$

$$(2)^2 = 2^4 \Rightarrow (2)^2 = 2^4$$

$$\sqrt{a+b} = u \quad \sqrt{a-b} = v \quad u-v = 2 \quad u^2 - v^2 = a+b - (a-b) = 2b$$

$$\Rightarrow u^2 - v^2 = (u+v)(u-v) = 2b \quad u+v = \frac{2b}{2} = b$$

$$\sqrt{a+b} + \sqrt{a-b} = 2 = u+v-2 \Rightarrow \frac{a+b}{2} - 2 = \frac{a-b}{2} - 2$$

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$$\left(\frac{a}{r}\right)$$