باران [illegible] . تکلیف ۲۱ . دهم ریاضی ۲

۱)

$(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+24!)$

$1+6+0+\dots+0=7 \times \frac{2+24+0+\dots+0}{6}$ $\quad 7\times 6=4\,2$

باران [illegible]

۲)

الف) $\sqrt[4]{(4+\sqrt7)^{-1}}\cdot\sqrt[2\times2]{(1+\sqrt7)^2}\Rightarrow$

$\sqrt[4]{\frac{1}{\cancel{4+\sqrt7}}\times\frac{2(\cancel{4+\sqrt7})}{1}}=\sqrt[4]{2}$

ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)$

$\frac{\sqrt2+\sqrt5}{\sqrt2(\sqrt5+\sqrt2)}=\frac{1}{\sqrt2}$

$\sqrt{\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)^2}\Rightarrow$

$3-\cancel{\sqrt5}+3+\cancel{\sqrt5}-2(2)=$

$3+3-4=2\Rightarrow\sqrt2$

$\frac{1}{\sqrt2}\times\sqrt2=1$

۳)

الف) $\frac{2\sqrt2+3\sqrt3}{5-\sqrt6}\times\frac{5+\sqrt6}{5+\sqrt6}=\frac{(2\sqrt2+3\sqrt3)(5+\sqrt6)}{19}$

$\frac{19\sqrt2+19\sqrt3}{19}=\sqrt2+\cancel{\sqrt3}-\cancel{\sqrt3}-1=$

$2\times\frac{1}{\sqrt3-1}\times\frac{\sqrt3+1}{\sqrt3+1}=\frac{\sqrt3+1}{\cancel{2}}\times\cancel{2}=\sqrt3+1$

$\sqrt2-1$

ب) $\frac{3\sqrt3-1}{4+\sqrt3}\times\frac{4-\sqrt3}{4-\sqrt3}=\frac{13\sqrt3-13}{13}=\sqrt3-1$

$\frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3}=\frac{2+\sqrt3}{1}$

$\sqrt3-1+2+\sqrt3=2\sqrt3+1$

۴)

الف) $\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}=\frac{\sqrt{(\sqrt3+1)+(\sqrt3-1)^2}}{\sqrt{\sqrt3-\sqrt2}}\Rightarrow$

$\frac{\sqrt{2\sqrt3+2\sqrt2}}{\sqrt{\sqrt3-\sqrt2}}\times\frac{\sqrt{\sqrt3+\sqrt2}}{\sqrt{\sqrt3+\sqrt2}}=$

درست حل

$\frac{\sqrt{2(\sqrt3+\sqrt2)^2}}{1}=\frac{\sqrt{(10+4\sqrt6)^2}}{1}=2+\sqrt6\,\boxed{-2}=$

$\sqrt6$

ب) $\sqrt[3]{2^2}\times\sqrt[4]{9^2\times2}\times\sqrt[12]{2^{12}}=$

$\sqrt[12]{2^8\times3^{12}\times2^3\times2^{12}}=\sqrt[12]{3^{12}\times2^{24}}$

$\Rightarrow 3\times4=12$

۵)

$\frac{2^n(1+3+9+27+81+243)}{2^{n-2}(1+2+4+8+16+32)}=81\Rightarrow 2^n(\cancel{364})=2^{n-2}(\cancel{3}\times 9)$

$\Rightarrow 2^{n-2}=2^{n-1}$

$n-2=0$

$n=2$

$$\left(\left(\sqrt{\sqrt{6}+2}+\sqrt{\sqrt{6}-2}\right)-2\sqrt[4]{3}\right)\left(\left(\sqrt{\sqrt{6}+2}+\sqrt{\sqrt{6}-2}\right)+2\sqrt[4]{3}\right)^2 \Rightarrow$$

$$\left(\sqrt{\sqrt{6}+2}+\sqrt{\sqrt{6}-2}\right)^2-4\sqrt{3} \Rightarrow \sqrt{6}+\cancel{2}+\sqrt{6}-\cancel{2}+2\sqrt{2}-4\sqrt{3}=\left(2\sqrt{6}-2\sqrt{3}\right)^2$$ (۶)

$24+8=\cancel{32}\,24-8\sqrt{12}$

$\cancel{36}-16\sqrt{3}=-16(2-\sqrt{3})$

$t=-16$

$A=2^{\frac{2}{5}}\times 2^{\frac{4}{15}}\times 2^{\frac{4}{3}}=2^{\frac{30}{15}}=2^{2}$

$(2\times 2^{2})^{-\frac{1}{3}}(2^{3})^{-\frac{1}{3}}=$

$2^{-1}=\frac{1}{2}$ ~~۲~~ (۸)

گزینه ۱

$\sqrt[7]{a}=a^{\frac{1}{7}}=27a^{\frac{15}{7}} \Rightarrow a^{-2}=27 \quad a=\frac{1}{\sqrt{27}}$ ، گزینه ۴

$\frac{3\sqrt{3}-3}{1+\sqrt{3}}=\frac{3(\sqrt{3}-1)}{\sqrt{3}+1}=-3$ ~~۴~~ (۹)

$a=\sqrt[42]{(2-\sqrt{3})^{21}}=\sqrt{2-\sqrt{3}}$: گزینه ۷ (۷)

$\left(\left(a+\frac{1}{a}\right)^2-2\right)^2=\left(a^2+\frac{1}{a^2}+2-2\right)^2=\left(a^2+\frac{1}{a^2}\right)^2 \Rightarrow$ (۸)

$2-\sqrt{3}+\frac{1}{2-\sqrt{3}}=2-\cancel{\sqrt{3}}+2+\cancel{\sqrt{3}}=4$

$\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{2+\sqrt{3}}{1}=(2+\sqrt{3})$

$(4)^2=2^4$ $\quad t=2^4$

$\sqrt{x+a}=u \quad \sqrt{x-4}=v \quad u-v=2 \quad u^2-v^2=\cancel{x}+a-\cancel{x}+4=a+4$

$\Rightarrow u^2-v^2=(u+v)(u-v)=a+4 \quad u+v=\frac{a+4}{2} \Rightarrow$ (۱)

$\sqrt{x+a}+\sqrt{x-4}-2=u+v-2 \Rightarrow \frac{a+4}{2}-2=\frac{a+4-4}{2}=$

$\frac{a}{2}$