پیمان قراگامی تکلیف ۲۱ زهرا رمضان

۲۵

۱)

$$(1!+3!+5!+\dots+25!)\,(2!+4!+6!+\dots+26!)$$

$$1+6=7 \qquad \text{صفر}$$

$$2+4 \to 6 \qquad \text{صفر}$$

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$$6\times 7 \Rightarrow \boxed{2}$$

۲)

الف) $\sqrt[4]{(4+\sqrt7)^{-1}}\,\sqrt{1+\sqrt7}$

$$\sqrt[4]{\frac{1}{4+\sqrt7}}\times\sqrt[4]{(1+\sqrt7)^2}\to\sqrt[4]{\frac{1}{4+\sqrt7}}\times\sqrt[4]{(1+\sqrt7)^2}=\sqrt[4]{2(4+\sqrt7)}$$

$$=\boxed{\sqrt[4]{2}}$$

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ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)\to\left(\frac{1}{\sqrt2}\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)\right)^2$

$$\sqrt{\frac12\left(3-\sqrt5+3+\sqrt5-4\right)}=-\sqrt1=\boxed{-1}$$

۳)

الف) $\frac{2\sqrt2+3\sqrt3}{5-\sqrt6}\times\frac{5+\sqrt6}{5+\sqrt6}=\frac{\cancel{19}(\sqrt2+\sqrt3)}{\cancel{10\sqrt2+4\sqrt3+15\sqrt3+9\sqrt2}}$

$$\cancel{19}$$

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ج) $\sqrt2+\sqrt3-\cancel{2}\left(\frac{1}{\cancel{\sqrt3-1}}\right)\times\frac{\sqrt3+1}{\cancel{\sqrt3+1}}\to\sqrt2+\cancel{\sqrt3}-\sqrt3-1=\boxed{\sqrt2-1}$

ب) $\frac{\sqrt{27}-1}{4+\sqrt3}+(2-\sqrt3)^{-1}\to\frac{3\sqrt3-1}{4+\sqrt3}\times\frac{4-\sqrt3}{4-\sqrt3}=$

$$\frac{\cancel{13}(12\sqrt3+\sqrt3-4-9)}{\cancel{13}}=\sqrt3-1+\left(\frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3}\right)=1+2\sqrt3$$

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الف) $\underbrace{\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}}_{A} - 2$

$A^2 = \frac{1+\sqrt{3}+\sqrt{3}-1+2\sqrt{(1+\sqrt{3})(\sqrt{3}-1)}}{\sqrt{3}-\sqrt{2}} =$

$\frac{2\sqrt{3}+2\sqrt{2}}{\sqrt{3}-\sqrt{2}} \overset{\text{مزدوج}}{\times} \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{2(\sqrt{3}+\sqrt{2})^2}{\cancel{3-2}_{1}}$

$A = \sqrt{2(\sqrt{3}+\sqrt{2})^2} \to \sqrt{2}(\sqrt{3}+\sqrt{2})$

$\sqrt{6}+2$

$\sqrt{6+2}-2 = \boxed{\sqrt{6}}$

ب) $\sqrt[12]{2^9} \times 3\sqrt[12]{2^3} \times \sqrt[12]{2^{12}}$ ۲

$3 \times \sqrt[12]{2^{24}} = 3 \times 2^2 = \boxed{12}$

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$\to \frac{3^n(1+3+9+27+81+243)}{2^{n-2}(1+2+2^2+2^3+2^4+2^5)} = 52$

$3^n \times 364 = 2^{n-2} \times \overset{9}{\cancel{63}} \times 52$ ۲

$3^n = 2^{n-2} \times 3^2$

$\to n-2 = 0 \to n = 2$

$\to n = 2$

$\Rightarrow \boxed{n = 2}$

6

$$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2 = t(2-\sqrt{3})$$

$$\left[\underbrace{(a^2+b^2-2ab)(a^2+b^2+2ab)}_{(a-b)^2(a+b)^2}\right]^2 \to \left[\underbrace{\cancel{(a-b)(a+b)}}_{a^2-b^2}\right]^2 \xrightarrow{\text{جایگذاری}} \boxed{16}$$

$$\underbrace{(a-b)^2(a+b)^2}_{((a-b)(a+b))^2}$$

$$\underbrace{\left(\left(\sqrt{\sqrt6-2}-\sqrt{\sqrt6+2}\right)^2\right)^2}_{\sqrt6-2+\sqrt6+2-2\sqrt{6-4}}$$

$$(2\sqrt6-2\sqrt2)^2 \to 24-8-8\sqrt{12}$$

7

$$a = \sqrt[4]{7-4\sqrt3} \to \sqrt[4]{(2-\sqrt3)^2} \to \sqrt{2-\sqrt3}$$

$$\left(\overbrace{\left(a+\frac1a\right)}^{x}+\sqrt2\right)^2\left(\left(a+\frac1a\right)-\sqrt2\right)^2 = 2^t$$

$$\left[\underbrace{(x+\sqrt2)(x-\sqrt2)}_{x^2-2}\right]^2 \to \left[a^2+\frac1{a^2}\right]^2 \to a^4+\frac1{a^4}+2 \xrightarrow{\text{جایگذاری}}$$

$$a^2+\frac1{a^2}+\cancel{2-2}$$

$$16 = 2^t$$

$$\boxed{t=4}$$

$$7-4\sqrt3+\frac{1}{7-4\sqrt3}+2 \to \boxed{16}$$

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$A = \sqrt[۵]{۴^۹ \, ۲^{۱۲}} \times \left(\frac{۱}{۲}\right)^{-\frac{۴}{۳}}$

$\sqrt[۱۵]{۴^۹ \times ۲^{۱۲}} \rightarrow \sqrt[۱۵]{۲^{۳۰}} = ۲^۲ = ۴$ ۲ ✓

$(۲^۹ \times ۴)^{-\frac{۱}{۳}} \rightarrow \left(\frac{۱}{۸}\right)^{\frac{۱}{۳}} \rightarrow \sqrt[۳]{\frac{۱}{۸}} = \frac{۱}{۲}$

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$\sqrt[۷]{a} = ۲۷ \times \sqrt[۷]{a^۵}$

$\frac{\sqrt[۷]{a}}{\sqrt[۷]{a^{۵}}} = ۲۷ \rightarrow \sqrt[۷]{\frac{a^۱}{a^۵}} = \sqrt[۷]{a^{-۴}} = ۲۷$

$\frac{۳\sqrt{۳} - ۳}{۱ + \sqrt{۳}} \times \frac{۱ - \sqrt{۳}}{۱ - \sqrt{۳}} = \boxed{-۳\sqrt{۳} + ۶}$ ۲ ✓

$a^{-۲} = ۲۷$

$\frac{۱}{a^۲} = ۲۷ \rightarrow \frac{۱}{a} = ۳\sqrt{۳}$

۱۰

$\underbrace{\sqrt{x+a} + \sqrt{x-۴}}_{۲} = ۴$ $\sqrt{x+a} - \sqrt{x-۴} = ۲$

$\rightarrow \frac{(\sqrt{x+a} - \sqrt{x-۴})(\sqrt{x+a} + \sqrt{x-۴})}{۲} =$ ۲ ✓

$\frac{x + a - (x - ۴)}{۲} = \frac{a + ۴}{۲} - ۲ \quad \boxed{\frac{a + \cancel{۴} - \cancel{۴}}{۲} = \frac{a}{۲}}$