

* از این دو عدد از آن به علت وجود عامل ۲، رقم یکان مغزنی شود پس :

$$(1! + 3!)(2! + 4!) = (1+2)(2+2^2) = 3 \times 6 = 18 \rightarrow \boxed{\text{رقم یکان} = 8}$$

$\text{مارت} \rightarrow \text{رقم یکان} = 7$

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$$\sqrt[4]{\frac{1}{2+\sqrt{2}}} \times \sqrt[4]{(1+\sqrt{2})^2} = \sqrt[4]{\frac{1-\sqrt{2}}{2}} \times \sqrt[4]{\frac{1+\sqrt{2}}{2}} = \sqrt[4]{\frac{(1-\sqrt{2})(1+\sqrt{2})}{4}} = \sqrt[4]{\frac{1-2}{4}} = \sqrt[4]{\frac{-1}{4}} = \sqrt[4]{-1} \times \sqrt[4]{\frac{1}{4}} = \sqrt[4]{-1} \times \frac{1}{\sqrt{2}}$$

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$$\text{ب) } \left(\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}(\sqrt{3}+\sqrt{2})} \right) = \frac{1}{\sqrt{2}} \text{ ①} \rightarrow \text{①, ②} \rightarrow \frac{1}{\sqrt{2}} \times -\sqrt{2} = -1$$

$$(\sqrt{3}-\sqrt{5}-\sqrt{3+\sqrt{5}}) \rightarrow t < 0 \rightarrow t^2 = 3 - \sqrt{5} + 3 + \sqrt{5} - 2\sqrt{3-\sqrt{5}} = 6 - 2\sqrt{3-\sqrt{5}} = 2 \rightarrow |t| = \sqrt{2} \text{ ②}$$

$$\text{الف) } \frac{2\sqrt{2}+3\sqrt{3}}{5-\sqrt{2}} \times \frac{5+\sqrt{2}}{5+\sqrt{2}} = \frac{10\sqrt{2}+3\sqrt{3}+15\sqrt{2}+3\sqrt{6}}{25-2} = \frac{25\sqrt{2}+3\sqrt{3}+3\sqrt{6}}{23} = \frac{25(\sqrt{2}+\sqrt{3})}{23} = \sqrt{2}+\sqrt{3} \text{ ①}$$

$$2 \left(\frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \right) = 2 \left(\frac{\sqrt{3}+1}{3-1} \right) = \sqrt{3}+1 \text{ ②} \rightarrow \text{①, ②: } (\sqrt{3}+\sqrt{2}) - (\sqrt{3}+1) = \sqrt{2}-1$$

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$$\text{ب) } \frac{4\sqrt{3}-1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{8\sqrt{3}-2-2\sqrt{3}+1}{4-3} = \frac{6\sqrt{3}-1}{1} = 6\sqrt{3}-1 \text{ ①}$$

$$\frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3} \text{ ②}$$

$$\left\{ \begin{array}{l} \text{①} \rightarrow \sqrt{3}-1+2+\sqrt{3} \\ = 2\sqrt{3}+1 \end{array} \right. \checkmark$$

$$\text{الف) } \left(\frac{\sqrt{3}+1+\sqrt{3}-1}{\sqrt{3}-2} \right)^2 \rightarrow \frac{2\sqrt{3}}{\sqrt{3}-2} = \frac{2\sqrt{3} \times (\sqrt{3}+2)}{(\sqrt{3}-2)(\sqrt{3}+2)} = \frac{2(3+2\sqrt{3})}{3-4} = -2(3+2\sqrt{3}) = -6-4\sqrt{3}$$

$$t^2 = 2(3+2\sqrt{3}) \rightarrow t = \sqrt{2(3+2\sqrt{3})} = \sqrt{4+4\sqrt{3}} = 2\sqrt{1+\sqrt{3}} \rightarrow t-2 = \sqrt{4+4\sqrt{3}}-2 = \sqrt{4} = 2$$

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$$\text{ب) } 2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 2^{\frac{1}{6}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{6}} = 2^{\frac{1}{2}+\frac{1}{6}+\frac{1}{3}+\frac{1}{6}+\frac{1}{6}} \times 3^{\frac{1}{3}} = 2^{\frac{5}{6}+\frac{1}{3}} \times 3^{\frac{1}{3}} = 2^{\frac{7}{6}} \times 3^{\frac{1}{3}} = 2 \times 3 = 6$$

$$\frac{3^{\frac{1}{2}} \left(1+3+9+\dots+3^{\infty} \right)}{2^{\frac{1}{2}-2} \left(1+2+4+\dots+2^{\infty} \right)} = \frac{3^{\frac{1}{2}} \left(\frac{3^{\frac{1}{2}}-1}{3^{\frac{1}{2}}-1} \right)}{2^{\frac{1}{2}-2} \left(\frac{2^{\frac{1}{2}}-1}{2^{\frac{1}{2}}-1} \right)} = 3^{\frac{1}{2}} \times 2^{\frac{3}{2}} = (3 \times 2)^{\frac{1}{2}} = \sqrt{6}$$

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$$\text{رنباله دهنه سی : } q=3, a=1$$

$$\text{رنباله دهنه سی : } q=2, a=1$$

$$\rightarrow \left(\frac{3}{2} \right)^{\frac{1}{2}} = \frac{q}{p} \rightarrow \boxed{\frac{1}{2} = \frac{1}{2}} \rightarrow \boxed{\frac{1}{2} = \frac{1}{2}}$$

$$\begin{aligned} & \rightarrow (\sqrt{r-r} + \sqrt{r+r})^r \\ & ((a^r + b^r)^r - (rab)^r)^r = (\sqrt{r} - (\sqrt{r} + \sqrt{r} + r - r) = (r\sqrt{r} - r(\sqrt{r}))^r = (r - \sqrt{r}) = \frac{r}{r\sqrt{r} + 1} - 14\sqrt{r} = \frac{1}{r} (r - \sqrt{r}) \\ & \rightarrow 14(r - \sqrt{r}) = \frac{1}{r} (r - \sqrt{r}) \rightarrow \boxed{\frac{1}{r} = 14} \end{aligned}$$

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$$C_{11} = \left(\left(a + \frac{1}{a} \right)^r - r \right)^r = \left(a^r + \frac{1}{a^r} \right)^r = a^r + \frac{1}{a^r} + r$$

$$C_{12} = \frac{1}{\sqrt{r} - r\sqrt{r}} \times \frac{\sqrt{r} + r\sqrt{r}}{\sqrt{r} + r\sqrt{r}} = \frac{\sqrt{r} + r\sqrt{r}}{r^2 - r} = \sqrt{r} + r\sqrt{r}$$

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$$a^r + \frac{1}{a^r} + r = \sqrt{r} - r\sqrt{r} + \sqrt{r} + r\sqrt{r} + r = 14 \rightarrow r^{\frac{1}{r}} = \frac{14}{r} \rightarrow \boxed{\frac{1}{r} = r}$$

$$A = r^{\frac{r}{r}} \times r^{\frac{r}{r}} \times r^{\frac{r}{r}} = r^r = r$$

$$(rA)^{-\frac{1}{r}} = (r \times r)^{-\frac{1}{r}} = (r^2)^{-\frac{1}{r}} = (r^r)^{-\frac{1}{r}} = \boxed{r^{-1} \times \frac{1}{r}}$$

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$$\begin{aligned} \sqrt{a} &= r\sqrt{a} \frac{1}{\sqrt{r}} \rightarrow a^{\frac{1}{r}} = r^r \times a^{\frac{1}{r}} \rightarrow r^r = \frac{a^{\frac{1}{r}}}{a^{\frac{1}{r}}} = r^r = a^{\frac{1}{r}} \rightarrow r^r = a^{\frac{1}{r}} \rightarrow a^{\frac{1}{r}} = \frac{1}{r\sqrt{r}} \\ \rightarrow \frac{1}{a} &= r\sqrt{r} \rightarrow \frac{1}{a} = r \rightarrow r\sqrt{r} = r \rightarrow r(\sqrt{r} - 1) \times \frac{1 - \sqrt{r}}{1 - \sqrt{r}} = \frac{r(\sqrt{r} - 1 - r + r\sqrt{r})}{-r} \\ &\Rightarrow \boxed{-r(\sqrt{r} + 1)} \end{aligned}$$

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$$\begin{aligned} & \frac{\sqrt{x+a}}{A} + \frac{\sqrt{x-r}}{B} = r = ? \quad \frac{\sqrt{x+a}}{A} - \frac{\sqrt{x-r}}{B} = r \\ & \left\{ \begin{aligned} A+B &= r = ? \\ A-B &= r \end{aligned} \right. \rightarrow \frac{A+B}{2} = r \rightarrow A+B = 2r \rightarrow A = 2r - B \\ & \text{① } B+r+B-r = rB = ? \quad \text{② } r \times \frac{a}{r} = \frac{a}{r} \\ & \text{③ } rB = x+a-x+r-r = r \\ & \text{④ } B = \frac{a}{r} \quad \text{⑤} \end{aligned}$$

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